Optimal Sensing Cardinality for Cognitive Radios

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***Abstract*—This letter puts forth a proactive approach for determining the sensing cardinality (the number of subchannels to sense) in a multiband cognitive radio network with limited available block energy. Specifically, we present the sensing cardinality that maximizes the achievable block rate of the cognitive user by optimally allocating available energy to the wideband spectrum sensing and data transmission. We further derive the closed-form expression for this optimal sensing cardinality when each subchannel has the identical primary spectrum occupancy statistics.**

***Index Terms*—Cognitive radio, opportunistic spectrum access, sensing cardinality.**

I. Introduction

SPECTRUM-AGILE radios have drawn considerable attention to address the spectrum scarcity problem in wireless communications [1]. One way to tackle this spectrum scarcity problem is to employ opportunistic spectrum access (OSA) for secondary reuse of the under-utilized spectrum. The OSA requires a spectrum sensing procedure, which consumes energy, to determine the primary channel occupancy state. This may be energy-inefficient or even infeasible, due to hardware limitations, to sense all available channels if a large number of channels is open to opportunistically access. Therefore, in energy-limited applications, the cognitive user must resourcefully utilize available energy for the spectrum sensing and data transmission [2]. Recently, spectrum sensing with multiple antennas was investigated for the energy detection (ED) and generalized likelihood ratio (GLR) sensing algorithms [3], [4].

In this letter, we develop a *proactive* approach for determining the optimal number of subchannels to sense, called the sensing cardinality, which maximizes the achievable block rate of the secondary user with limited energy in a multiband cognitive radio network. Using the optimal sensing cardinality, the secondary user allocates the available energy to the spectrum sensing and data transmission to maximize its achievable block rate. In case of identical primary spectrum occupancy statistics over all frequency subchannels, we further determine this optimal sensing cardinality in closed form.

II. System Model

We consider an energy-limited multiband cognitive radio network. The channel consists of N_{sub} frequency bands (subchannels), each has bandwidth B_{c} and experiences independent and identically distributed (i.i.d.) flat Rayleigh fading. The fading coefficients of each subchannel remain constant within a block of coherence time T_{c} and change independently from one block to the other. Using a hierarchical access structure with spectrum overlay, primary users have licensed access and secondary users have opportunistic access to each subchannel. We model the primary spectrum occupancy in each subchannel according to an independent Bernoulli process with two states, namely, "idle" and "busy" states, where the occupancy state is kept fixed in each coherence block and changes independently in the next block, as considered in [5] and [6]. Let q_i be the probability that the ith subchannel is in the busy state for each time block, which is known *a priori* to the secondary user. We rearrange the statistics q_i's in increasing order as $\mathsf{q}_{[1]} \leq \mathsf{q}_{[2]} \leq \ldots \leq \mathsf{q}_{[N_{\mathrm{sub}}]}$.

In each coherence block, the secondary user chooses a set of best subchannels (in terms of the statistics q_i) to sense, which is referred to as the *wideband spectrum sensing set* $\mathcal{S} \subseteq \{1, 2, \ldots, N_{\mathrm{sub}}\}$ and its cardinality $|\mathcal{S}|$ as the *sensing cardinality*. After sensing these subchannels simultaneously requiring energy $\mathcal{E}_{\mathrm{s}}$, the secondary user determines a set of subchannels sensed as idle, denoted by $\mathcal{S}_{\mathrm{idle}} \subseteq \mathcal{S}$, and chooses a subchannel with the smallest q_i in $\mathcal{S}_{\mathrm{idle}}$ for data transmission.[1]

To prolong secondary network lifetime and to limit network interference caused by the secondary users, the energy $\mathcal{E}_{\mathrm{tot}}$ per coherence block is dedicated for both sensing and data transmission. The secondary user attempts to optimally allocate the available energy $\mathcal{E}_{\mathrm{tot}}$ to the spectrum sensing and data transmission. The secondary user transmits data with power $\mathcal{P}$ over the chosen subchannel i to the secondary receiver. The received signal over this subchannel can be expressed as

$$Y_i = \begin{cases} \sqrt{\gamma_{\mathrm{s}}} H_i X_{\mathrm{s}} + Z_i, & \text{if the subchannel is idle,} \\ \sqrt{\gamma_{\mathrm{s}}} H_i X_{\mathrm{s}} + \sqrt{\gamma_{\mathrm{p}}} G_i X_{\mathrm{p}} + Z_i, & \text{otherwise,} \end{cases} \tag{1}$$

where X_{s} and X_{p} are the transmitted signals from the secondary and primary users with $\mathbb{E}\left\{|X_{\mathrm{s}}|^2\right\} = \mathbb{E}\left\{|X_{\mathrm{p}}|^2\right\} = 1$; H_i and G_i are channel coefficients from the secondary and primary users to the secondary receiver; and Z_i is the complex additive white Gaussian noise. All the H_i, G_i, and Z_i are i.i.d. circularly symmetric complex Gaussian with zero mean and unit variance. The quantities γ_{s} and γ_{p} denote the average received signal-to-noise ratios (SNRs) for the secondary and primary signals, respectively.

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[1] See, e.g., [7]–[9] for a wideband spectrum sensing framework. To identify the set $\mathcal{S}_{\mathrm{idle}}$ of idle subchannels, the radio front-end can employ a bank of narrowband bandpass filters followed by the detector (ED or GLR) for each narrowband subchannel [7], [8]. Using the irregularity of the power spectral density, the wavelet detection can also be employed for effective wideband spectrum sensing [9].

III. Optimal Sensing Cardinality

In terms of the available energy $\mathcal{E}_{\rm tot}$ and transmit power $\mathcal{P}$, we denote the maximum data transmission time of the secondary user by $\mathcal{T}_{\max} \triangleq \mathcal{E}_{\rm tot}/\mathcal{P}$. This corresponds to the transmission time duration for the case that the secondary user aggressively accesses to the subchannel without sensing and uses the available energy $\mathcal{E}_{\rm tot}$ fully for data transmission with power $\mathcal{P}$. For energy-limited applications, we assume that $\mathcal{T}_{\max} < T_{\rm c}$. Let $\mathcal{E}_0$ be the sensing energy cost per subchannel, called the *per-subchannel sensing energy*, and $\zeta \triangleq \mathcal{E}_0/\mathcal{E}_{\rm tot}$ be the ratio between the per-subchannel sensing energy and block energy, called the *sensing energy figure*, which reflects the energy efficiency of a sensing procedure. To determine the optimal sensing cardinality, we develop a proactive approach requiring only knowledge of the primary occupancy statistics and the spectrum sensing effectiveness such as the sensing energy figure ζ and the operating characteristics (false alarm and miss detection probabilities).

Let $\mathsf{p}_{\rm f}$ and $\mathsf{p}_{\rm m}$ be the false alarm and miss detection probabilities of the secondary user for detecting the primary spectrum occupancy in each subchannel. Then, the probability that the ith subchannel is sensed to be idle is given by

$$\mathsf{p}_i\left(\text{idle}\right) = \left(1-\mathsf{p}_{\rm f}\right)\left(1-\mathsf{q}_i\right) + \mathsf{p}_{\rm m}\mathsf{q}_i. \tag{2}$$

The achievable rate in nats/s/Hz of the secondary user over the ith subchannel, $i \in \mathcal{S}_{\rm idle}$, is given by

$$C_i = \left(1-\alpha_i\right)\ln\left(1+\gamma_{\rm s}|H_i|^2\right) + \alpha_i \ln\left(1+\frac{\gamma_{\rm s}|H_i|^2}{1+\gamma_{\rm p}|G_i|^2}\right) \tag{3}$$

where $\alpha_i = \mathsf{p}_{\rm m}\mathsf{q}_i/\mathsf{p}_i\left(\text{idle}\right)$ denotes the probability that the ith subchannel is occupied by the primary user, conditioned on the event that the secondary user senses this subchannel to be idle.

Lemma 1: The achievable average block rate in nats/Hz per block of the secondary user with available energy $\mathcal{E}_{\rm tot}$ is

$$R\left(\zeta, |\mathcal{S}|\right) = \mathcal{T}_{\max}\left(1-\zeta|\mathcal{S}|\right)K_{|\mathcal{S}|} \tag{4}$$

with

$$K_{|\mathcal{S}|} = \sum_{k=1}^{|\mathcal{S}|}\left(\prod_{i=1}^{k-1}\mathsf{p}_{[i]}\left(\text{busy}\right)\mathsf{p}_{[k]}\left(\text{idle}\right)\right)\langle C_{[k]}\rangle \tag{5}$$

where $\mathsf{p}_i\left(\text{busy}\right) = 1-\mathsf{p}_i\left(\text{idle}\right)$ and[2]

$$\langle C_i\rangle = e^{\frac{1}{\gamma_{\rm s}}}\left(1+\frac{\alpha_i\gamma_{\rm p}}{\gamma_{\rm s}-\gamma_{\rm p}}\right)E_1\left(\frac{1}{\gamma_{\rm s}}\right) - e^{\frac{1}{\gamma_{\rm p}}}\frac{\alpha_i\gamma_{\rm s}}{\gamma_{\rm s}-\gamma_{\rm p}}E_1\left(\frac{1}{\gamma_{\rm p}}\right). \tag{6}$$

Proof: Since $|H_i|^2$ and $|G_i|^2$ obey the exponential distribution with unit mean, the secondary user can achieve the average rate $\langle C_i\rangle = \mathbb{E}_{H_i,G_i}\left\{C_i\right\}$ as in (6) for chosen subchannel $i \in \mathcal{S}_{\rm idle}$. Furthermore, the secondary user consumes the sensing energy $\mathcal{E}_{\rm s}$ equal to $\mathcal{E}_0|\mathcal{S}|$ and uses the remaining part of $\mathcal{E}_{\rm tot}$ for data transmission, leading to $\left(\mathcal{E}_{\rm tot}-\mathcal{E}_{\rm s}\right)/\mathcal{P} = \mathcal{T}_{\max}\left(1-\zeta|\mathcal{S}|\right)$, from which we complete the proof. $\square$

[2] $E_n\left(z\right)$ is the exponential integral function of order n.

Theorem 1: Let $\mathcal{L} = \{1, 2, \ldots, t_{\max} = \min\left(\lfloor 1/\zeta\rfloor, N_{\rm sub}\right)\}$ be a set of feasible sensing cardinalities.[3] Then, $R\left(\zeta, |\mathcal{S}|\right)$ is a concave function in $|\mathcal{S}| \in \mathcal{L}$ and hence, there exists the unique optimal sensing cardinality that maximizes $R\left(\zeta, |\mathcal{S}|\right)$ such that

$$|\mathcal{S}|^\star = \arg\max_{|\mathcal{S}|\in\mathcal{L}}\left(1-\zeta|\mathcal{S}|\right)K_{|\mathcal{S}|}, \tag{7}$$

which can be readily determined since $\mathcal{L}$ is a finite integer set.

Proof: Let $f\left(t\right)$ be the first-order interpolation of $K_{|\mathcal{S}|}$'s such that $f\left(t\right) = K_{|\mathcal{S}|} + \left(K_{|\mathcal{S}|+1} - K_{|\mathcal{S}|}\right)\left(t-|\mathcal{S}|\right)$ if $|\mathcal{S}| \le t \le |\mathcal{S}|+1$, $|\mathcal{S}| \in \mathcal{L}$. Since $K_{|\mathcal{S}|+1} + K_{|\mathcal{S}|-1} < 2K_{|\mathcal{S}|}$, it is easy to show that $f\left(t\right)$ is an increasing and concave function in $t \in [1, t_{\max}]$. Hence, since $\left(1-\zeta t\right)f\left(t\right)$ is also concave in $t \in [1, t_{\max}]$, $R\left(\zeta, |\mathcal{S}|\right)$ is a concave function in $|\mathcal{S}| \in \mathcal{L}$. $\square$

Corollary 1: Let $\mathsf{q}_i = \mathsf{q}$ and $\mathsf{p}_i\left(\text{busy}\right) = \mathsf{p}\left(\text{busy}\right)$ for all subchannels. Then, the optimal sensing cardinality that maximizes $R\left(\zeta, |\mathcal{S}|\right)$ is given in closed form as

$$|\mathcal{S}|^\star = \left\lfloor \frac{1}{\zeta} + \frac{W\left(e^{1-\zeta^{-1}\ln\mathsf{p}(\text{busy})}\right)-1}{\ln\mathsf{p}\left(\text{busy}\right)}\right\rceil \tag{8}$$

where $\lfloor t\rceil = \arg\max_{\Delta\in\{\lfloor t\rfloor, \lceil t\rceil\}}\left(1-\zeta\Delta\right)K_\Delta$ and $W\left(z\right)$ is the Lambert W-function.

Proof: Since all the subchannels have the same primary spectrum occupancy statistics q leading to the *homogeneous* capacity $\langle C_i\rangle = \langle C\rangle$, (5) reduces to

$$K_{|\mathcal{S}|} = \left(1-\mathsf{p}(\text{busy})^{|\mathcal{S}|}\right)\langle C\rangle. \tag{9}$$

Let $g\left(|\mathcal{S}|\right) = \left(1-\zeta|\mathcal{S}|\right)K_{|\mathcal{S}|}$. Then, since $1-\zeta|\mathcal{S}| \ge 0$ and $0 \le \mathsf{p}\left(\text{busy}\right) \le 1$, it is obvious that $g\left(t\right)$ is a concave function in $t \in [0, 1/\zeta]$. Since $\left[\partial g\left(t\right)/\partial t\right]\big|_{t=0} \ge 0$ and $\left[\partial g\left(t\right)/\partial t\right]\big|_{t=1/\zeta} \le 0$, there exists the optimal t that maximizes $g\left(t\right)$ over $t \in [0, 1/\zeta]$, which is the unique solution of $\partial g\left(t\right)/\partial t = 0$. Let

$$x = -t + \frac{1}{\zeta} - \frac{1}{\ln\mathsf{p}\left(\text{busy}\right)}. \tag{10}$$

Then, we have

$$\begin{aligned}
&\partial g\left(t\right)/\partial t = 0 \\
\Leftrightarrow\ & \mathsf{p}(\text{busy})^{-t} = \ln\mathsf{p}(\text{busy})^{t-1/\zeta} + 1 \\
\Leftrightarrow\ & x\,\mathsf{p}(\text{busy})^{-x} = -\frac{\mathsf{p}(\text{busy})^{1/\ln\mathsf{p}(\text{busy})-1/\zeta}}{\ln\mathsf{p}\left(\text{busy}\right)} \\
\Leftrightarrow\ & x = -\frac{W\left(\mathsf{p}(\text{busy})^{1/\ln\mathsf{p}(\text{busy})-1/\zeta}\right)}{\ln\mathsf{p}\left(\text{busy}\right)},
\end{aligned} \tag{11}$$

from which we can obtain the optimal value of t. Finally, restricting the optimal t to a positive integer and comparing $g\left(|\mathcal{S}|\right)$ at two tentative points $|\mathcal{S}| = \lfloor t\rfloor$ and $|\mathcal{S}| = \lceil t\rceil$, we obtain the optimal $|\mathcal{S}|^\star$ as in (8). $\square$

Remark 1: We can extend Lemma 1 and Theorem 1 (also Corollary 1) to a multiple-input multiple-output (MIMO) case by replacing $\langle C_i\rangle$ in (6) with the MIMO capacity, which can be evaluated in closed form using [10, Theorem II.1] and [11, Corollary 1].

[3] $\lfloor x\rfloor$ and $\lceil x\rceil$ denote the largest previous and the smallest following integers of x, respectively.

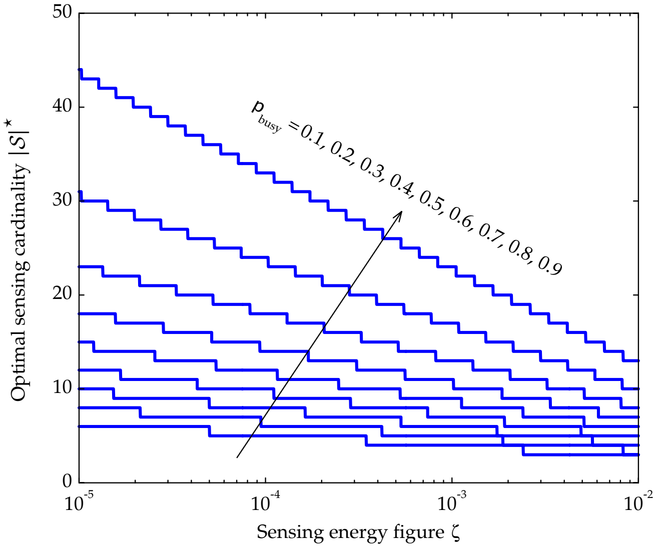

Fig. 1. Optimal sensing cardinality $|\mathcal{S}|^\star$ as a function of ζ.

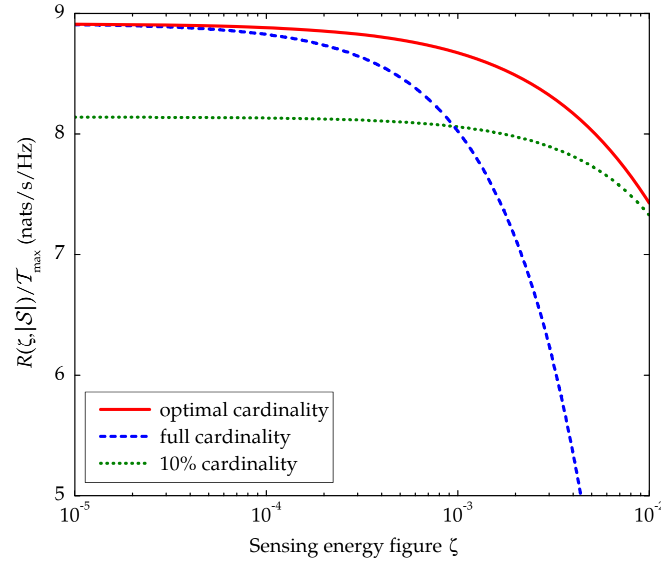

Fig. 2. $R(\zeta, |\mathcal{S}|)/\mathcal{T}_{\max}$ as a function of ζ at $\mathsf{p}_{\text{busy}} = 0.9$.

Remark 2: Since $K_{|\mathcal{S}|}$ is a function of $\langle C_i \rangle$, the optimal sensing cardinality $|\mathcal{S}|^\star$ depends on the number of antennas for the MIMO setup. In particular, for perfect sensing, $|\mathcal{S}|^\star$ relies solely on ζ and the primary spectrum occupancy statistics q_i's. Moreover, the MIMO antenna configuration also affects the sensing energy figure ζ and operating characteristics (p_f and p_m) for a specific sensing algorithm.[4]

IV. Results and Discussion

For numerical results, we set $N_{\text{sub}} = 100$, $\gamma_\text{s} = 15$ dB, $\gamma_\text{p} = 10$ dB, $\mathcal{E}_0 = 5\,000$ energy units, $\mathsf{p}_\text{m} = 0.1$, $\mathsf{p}_\text{f} = 0.1$, and $\mathsf{q}_i = 0.001\,(50 - i) + \mathsf{p}_{\text{busy}}$. Furthermore, we consider that all the terminals are equipped with three antennas. Figure 1 shows the optimal sensing cardinality $|\mathcal{S}|^\star$ as a function of ζ for various p_{busy} values ranging from 0.1 to 0.9. We can see that $|\mathcal{S}|^\star$ increases as the sensing energy figure ζ decreases and/or p_{busy} increases. This implies that the secondary user must sense more subchannels to increase the probability of finding an idle subchannel when ζ is small (energy-efficient spectrum sensing) and/or p_{busy} is large (dense primary spectrum occupancy). Figure 2 shows the normalized achievable average rate $R(\zeta, |\mathcal{S}|)/\mathcal{T}_{\max}$ as a function of ζ at $\mathsf{p}_{\text{busy}} = 0.9$ for the optimal sensing cardinality $|\mathcal{S}|^\star$, full cardinality ($|\mathcal{S}| = N_{\text{sub}}$), and 10% cardinality ($|\mathcal{S}| = 10$). We see that the use of optimal cardinality enables the secondary user to maximize the achievable rate by appropriately balancing energy consumption of the sensing and data transmission. As ζ increases, $R(\zeta, |\mathcal{S}|)/\mathcal{T}_{\max}$ decreases, especially for the case of full cardinality sensing, due to energy inefficiency of the sensing process. The full cardinality case overuses energy for sensing while the 10% cardinality case underuses the sensing energy, both leading to a significant rate loss due to improper energy usage.

The notion of optimal sensing cardinality can be underpinned by sophisticating an access decision-making strategy such as simultaneous multichannel access or best single-channel access chosen by probing a channel quality, primary occupancy history, and/or access history to maximize a rate while minimizing channel switching overhead and/or collisions with primary (or other secondary) transmissions. If the secondary user has no prior information on q_i's, we can consider their estimation phase before using the optimal $|\mathcal{S}|^\star$ as in [6].

[4]See, e.g., [3] where the required number of samples for sensing (which is *proportionally* related to ζ) is involved in the antenna number of the secondary user for certain values of p_f and p_m.

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