

Optimal Linear Multihop System for DF Relaying in a Poisson Field of Interferers

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Abstract—This paper provides a strategy for multihop transmission with decode-and-forward (DF) in a Poisson field of interferers. We analyze the optimal linear multihop system such as optimal resource allocation, optimal placement of the relay nodes, and optimal number of hops for multihop DF relaying which minimize their outage probability.

Index Terms—Linear multihop relaying, Poisson network, optimal placement of nodes, optimal number of hops

I. INTRODUCTION

RECENTLY, multihop relaying has been studied as an effective method for establishing connectivity between the nodes of a network where direct transmission is not feasible. In particular, many studies on how to find optimal routing for multihop relaying are found in the literature for a variety of scenarios [1]–[3]. In [1], the researchers assumed equidistant relay nodes in a linear multihop relay network and investigated the optimal number of hops for decode-and-forward (DF) relaying for only large scale fading channels in a bandwidth-limited regime. [2] studied the optimal number of hops using mean channel state information (CSI) assuming equal power allocation and proposed an efficient routing algorithm for cross-layer designs. More recently, [3] computed the optimal number of hops for linear multihop amplify-and-forward (AF) relaying with equal resource allocation in terms of random coding error exponents.

To advance the previous studies on relay networks focused on noise-limited fading environments, multihop relaying in the presence of co-channel interference has been studied in [4]–[6], but they still neglected either noise or interference at each node for analytical tractability and assumed that the location of network nodes are deterministic. Hence, [7]–[9] considered randomly distributed interferers in a Poisson field to maintain spatial randomness of each node for practical wireless ad hoc networks. In particular, [9] computed the optimal number of hops for a multihop transmission to minimize end-to-end delay considering the service and waiting times at the node buffers in a Poisson field of interference.

This paper considers a linear multihop transmission with the DF strategy in a Poisson field of interferers. Furthermore,

Manuscript received August 27, 2013. The associate editor coordinating the review of this letter and approving it for publication was N. Pappas.

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Digital Object Identifier 10.1109/LCOMM.2013.101413.131959

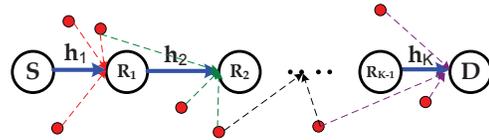


Fig. 1. Linear multihop relay network. The filled circles represent randomly distributed interferers and the dashed lines represent interference.

since interferers that form a Poisson point process (PPP) in each hop can be located at the same placement across the hops, we consider a spatial correlation in the interference. In a linear multihop network, an increase in the number of transmission hops degrades the performance of the multihop relaying due to a higher occupation of bandwidth or time and the waste of network resources. Many previous works to find the optimal number of hops made assumptions about system parameters such as transmission power, distance, and time fraction for analytical tractability [1]–[3]. For these reasons, we identify the optimal linear multihop system, such as optimal resource allocation, optimal placement of the relay nodes, and optimal number of hops, for multihop DF relaying to minimize their outage probability. Our results reveal that the multihop DF relaying with the optimal number of hops can achieve the minimum outage probability using equidistant relay nodes with the same transmission power and time fraction for each hop.

II. SYSTEM MODEL

We consider a K -hop wireless network consisting of a source node S and a destination node D , separated by a distance d_{SD} , and intermediate relay nodes R_1, R_2, \dots, R_{K-1} , as illustrated in Fig. 1. The source communicates with the destination by multihop transmission, where each relay node fully decodes the message and then transmits the re-encoded message to its respective successive node in orthogonal time-domain subchannels, called linear multihop DF relaying. Moreover, since the likelihood of scattered and uncoordinated network nodes grows as the number of network nodes increases rapidly, we assume randomly distributed transmitters which generate interference drawn from a homogeneous 2-D PPP of intensity λ , denoted Φ_{I_n} as the interferer set at the n -th hop.

From [8], [10], the received signal to interference-plus-noise ratio (SINR) at the n -th hop can be written as

$$\gamma_n = \frac{\mathcal{P}_n |h_n|^2}{\sum_{i \in \Phi_{I_n}} |g_i|^2 \mathcal{P}_{INF} + N_0}, \quad \text{for } n = 1, \dots, K \quad (1)$$

where \mathcal{P}_n denotes the transmission power of n -th hop, h_n is the channel between the transmitter and the receiver at the n -th hop, and N_0 is the variance of complex additive Gaussian noise at the n -th hop. Then, the channel strength of h_n is determined by both Rayleigh fading model and the large scale path loss following the power law with the path loss exponent α for a relatively lossy environment $2 < \alpha < 5$ in wireless networks. In this paper, we regard the aggregate of interference in Poisson field $\sum_{i \in \Phi_{I_n}} |g_i|^2 \mathcal{P}_{INF}$ as a shot noise [7]–[9] where a receiver is affected by its interferers located in a homogenous PPP Φ_{I_n} with the average transmit power of an interferer \mathcal{P}_{INF} . In addition, since the distributions in m -dimensional homogenous PPP is equivalent to the infinite network with uniformly random distribution, we consider a correlated interference due to the common interferer locations at each hop [11].

III. OPTIMAL LINEAR MULTIHOP SYSTEM

Before computing the optimal linear multihop system based on the outage probability, the cumulative density function (CDF) of SINR at the n -th hop in [10, eq. (14)] can be changed to consider the infinite interference region for a PPP and the general path loss exponent for aggregate of interference [11] as follows:

$$F_{\gamma_n}(z) = 1 - \exp \left[-\frac{d_n^\alpha N_0}{\mathcal{P}_n} z - \Delta \left(\frac{d_n^\alpha \mathcal{P}_{INF}}{\mathcal{P}_n} z \right)^\delta \right] \quad (2)$$

where $z > 0$, $\delta \triangleq 2/\alpha$, $\Delta \triangleq \lambda \pi \Gamma(1 - \delta) \Gamma(1 + \delta)$, and d_n is the distance of the n -th hop. Then, using the maximum average mutual information for the DF relay channel, we can obtain the outage probability of multihop DF relaying in the presence of correlated interference [11, eq. (2)] as

$$P_{out}(R) = \mathbb{P} \left\{ \min_{n=1, \dots, K} R_n < R \right\} = 1 - \prod_{n=1}^K \left[1 - F_{\gamma_n} \left(2^{\frac{R}{\beta_n}} - 1 \right) \right] = 1 - \exp \left[-\sum_{n=1}^K \left(\frac{d_n^\alpha N_0}{\mathcal{P}_n} \left(2^{\frac{R}{\beta_n}} - 1 \right) + \Delta D_n(\delta) \left(\frac{d_n^\alpha \mathcal{P}_{INF}}{\mathcal{P}_n} \left(2^{\frac{R}{\beta_n}} - 1 \right) \right)^\delta \right) \right] \quad (3)$$

where D_n is the polynomial of order $n-1$ defined as $D_n(x) = \frac{\Gamma(n+x)}{\Gamma(n)\Gamma(1+x)}$, R_n denotes the achievable rate of the n -th hop given by $R_n = \beta_n \log_2(1 + \gamma_n)$, β_n is the fraction of time for each hop, and R denotes the spectral efficiency.

A. Optimal Resource Allocation and Placement of the Relay Nodes for DF relaying

In this section, we compute the optimal resource allocation and placement of the relay nodes schemes to determine the transmission power \mathcal{P}_n , fraction of time β_n and distance d_n of each hop, which minimizes the outage probability of the multihop DF relaying (3).

The optimization problem based on minimizing the outage probability in (3) under some constraints can be expressed as

$$\arg \min_{\beta_n, \mathcal{P}_n, d_n} \sum_{n=1}^K \left(\frac{N_0 d_n^\alpha}{\mathcal{P}_n} \left(2^{\frac{R}{\beta_n}} - 1 \right) + \mathcal{Q} d_n^2 \left(\frac{2^{\frac{R}{\beta_n}} - 1}{\mathcal{P}_n} \right)^\delta \right)$$

subject to

$$\sum_{n=1}^K \beta_n \mathcal{P}_n = \mathcal{P}_T, \quad \sum_{n=1}^K \beta_n = 1, \quad \text{and} \quad \sum_{n=1}^K d_n = d_{SD} \quad (4)$$

where $\mathcal{Q} \triangleq \Delta D_n(\delta) (\mathcal{P}_{INF})^\delta$ and \mathcal{P}_T is the average transmission power of multihop relaying. Before computing the optimal resource allocation and placement of the relay nodes for multihop DF relaying, we prove that our optimization problem (4) is convex by using the positive definite of the matrix constructed by the second derivatives in Appendix A.

Then, as the optimization problem for multihop DF relaying is convex, it can be changed eliminating equality constraints in [12] as

$$\arg \min_{\mathcal{P}_n, d_n, \beta_n} f(\mathbf{P}, \mathbf{d}, \beta) = \arg \min_{\beta_n, \mathcal{P}_n, d_n} \left[\sum_{n=1}^{K-1} g(\mathcal{P}_n, d_n, \beta_n) + g \left(\frac{\mathcal{P}_T - \sum_{n=1}^{K-1} \beta_n \mathcal{P}_n}{1 - \sum_{n=1}^{K-1} \beta_n}, d_{SD} - \sum_{n=1}^{K-1} d_n, 1 - \sum_{n=1}^{K-1} \beta_n \right) \right] \quad (5)$$

where $g(\mathcal{P}_n, d_n, \beta_n) = \frac{N_0 \left(2^{\frac{R}{\beta_n}} - 1 \right)}{d_n^\alpha \mathcal{P}_n} + \mathcal{Q} d_n^2 \left(\frac{2^{\frac{R}{\beta_n}} - 1}{\mathcal{P}_n} \right)^\delta$. Applying the first-order optimality conditions, each \mathcal{P}_n , β_n and d_n for $n = 1, \dots, K$ must satisfy the following property:

$$\frac{\partial f(\mathbf{P}, \mathbf{d}, \beta)}{\partial \mathcal{P}_n} = \frac{\partial f(\mathbf{P}, \mathbf{d}, \beta)}{\partial \beta_n} = \frac{\partial f(\mathbf{P}, \mathbf{d}, \beta)}{\partial d_n} = 0. \quad (6)$$

Theorem 1 (Optimal resource allocation and placement):

The optimal resource allocation and placement of the relay nodes for the multihop DF relaying can be written as

$$\mathcal{P}_n = \mathcal{P}_T, \quad \beta_n = \frac{1}{K}, \quad d_n = \frac{d_{SD}}{K} \quad (7)$$

for $n = 1, \dots, K$ where β_K and d_K are the fraction of time and the distance of the K -th hop, respectively.

Proof: See Appendix B. ■

From the above results, for the optimal resource allocation in a linear multihop DF relay network, we transmit data with the same transmission power for each hop $\mathcal{P}_n = \mathcal{P}_T$ and use the same fraction of time for each hop $\beta_n = 1/K$. Furthermore, using each relay node deployed at the same distance in a direct line is the best way to maximize their performance from the optimal placement result.

B. Optimal Number of Hops for DF Relaying

In this section, we focus on the optimal number of hops K_{opt} for the multihop DF relaying to minimize their outage probability. The outage probability of the multihop DF relaying in the presence of correlated interference, obtained by applying the results of previous section $\beta_n = \frac{1}{K}$, $\mathcal{P}_n = \mathcal{P}_T$, and $d_n = d_{SD}/K$, can be expressed as

$$P_{out}(R) = 1 - \exp \left[- \left(\mathcal{N} K^{1-\alpha} \left(2^{KR} - 1 \right) + \mathcal{I} \left(2^{KR} - 1 \right)^\delta K^{-2} \sum_{n=1}^K D_n(\delta) \right) \right] \approx 1 - \exp \left[- \left(\mathcal{N} K^{1-\alpha} \left(2^{KR} - 1 \right) + c \mathcal{I} K^{\delta-1} \left(2^{KR} - 1 \right)^\delta \right) \right] \quad (8)$$

where $\mathcal{N} \triangleq \frac{d_{SD}^\alpha N_0}{\mathcal{P}_T}$ and $\mathcal{I} \triangleq \Delta \frac{d_{SD}^2}{\Gamma(1+\delta)} \left(\frac{\mathcal{P}_{INF}}{\mathcal{P}_T} \right)^\delta$. In particular, to compute K_{opt} , we use the asymptotic equality of $D_n(x)$ given by $D_n(x) \lesssim \frac{n^x}{\Gamma(1+x)}$ in [11, eq. (3)] and $\sum_{n=1}^K n^\delta \approx cK^{1+\delta}$ where c is a coefficient for approximation $0.5 < c < 1$.

Then, the optimization problem to minimize the outage probability is given by

$$K_{opt} = \arg \min_K f(K) \quad (9)$$

where

$$f(K) = \mathcal{N}K^{1-\alpha} (2^{KR} - 1) + c\mathcal{I}K^{\delta-1} (2^{KR} - 1)^\delta. \quad (10)$$

In this paper, we omit the proof of the convexity of this optimization problem, because $f''(K) > 0$ is straightforward.

However, since an exact solution of K_{opt} using the first-order optimality condition $f'(K) = 0$ is non-existent, we conjecture K_{opt} from upper and lower bounds using the sign of $f'(K)$ as follows:

Theorem 2 (Optimal number of hops for DF relaying):

The upper and lower bounds for the optimal number of transmission hops K_{opt} can be written as

$$\left[\frac{\frac{\alpha}{2} - 1 + W\left(-\frac{\frac{\alpha}{2}-1}{e^{\frac{\alpha}{2}-1}}\right)}{R \ln 2} \right]_+ \leq K_{opt} \leq \left[\frac{\alpha - 1 + W\left(-\frac{\alpha-1}{e^{\alpha-1}}\right)}{R \ln 2} \right]_+ \quad (11)$$

where $W(x)$ is the Lambert W function and $[x]_+$ denotes the positive integer closest to x .

Proof: From (10), $f'(K)$ can be written as

$$f'(K) = -\mathcal{N} \frac{K^{-\alpha} (\alpha - 1)}{(2^{KR} - 1)^{-1}} \underbrace{\left(1 - \frac{K(R \ln 2) 2^{KR}}{(\alpha - 1)(2^{KR} - 1)} \right)}_T - c\mathcal{I} \frac{K^{\delta-2} (1 - \delta)}{(2^{KR} - 1)^{-\delta}} \underbrace{\left(1 - \frac{K(R \ln 2) 2^{KR}}{(\alpha/2 - 1)(2^{KR} - 1)} \right)}_U. \quad (12)$$

For the lower bound, we compute $T > 0$ and $U > 0$ to satisfy $f'(K) < 0$ as

$$e^t [(\alpha - 1) - t] > \alpha - 1 \quad \text{for } T > 0, \quad (13)$$

$$e^u [(\alpha/2 - 1) - u] > \alpha/2 - 1 \quad \text{for } U > 0 \quad (14)$$

where $t = u = KR \ln 2$. From the above results, the lower bound is determined by the closest positive integer to satisfy $f'(K) < 0$. On the contrary, we obtain the upper bound for K_{opt} using $T < 0$ and $U < 0$ to satisfy $f'(K) > 0$. ■

Corollary 1: If the interference is independent across the overall hops, we would have

$$\left[\frac{\frac{\alpha}{2} + W\left(-\frac{\frac{\alpha}{2}}{e^{\frac{\alpha}{2}}}\right)}{R \ln 2} \right]_+ \leq K_{opt} \leq \left[\frac{\alpha - 1 + W\left(-\frac{\alpha-1}{e^{\alpha-1}}\right)}{R \ln 2} \right]_+ \quad (15)$$

Proof: As $f(K) = \mathcal{N}K^{1-\alpha} (2^{KR} - 1) + \mathcal{I}K^{-1} (2^{KR} - 1)^\delta$, we arrive at the desired result (15) easily. ■

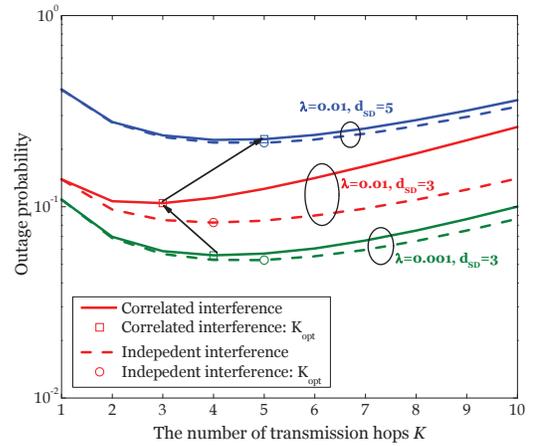


Fig. 2. Outage probability of the multihop DF relaying as a function of K for different $d_{SD} = 3$ and 5 and $\lambda = 0.001$ and 0.01 with $\mathcal{P}_T = 20$ dB, $\mathcal{P}_{INF} = 5$ dB, $\alpha = 3$, and $R = 0.5$. The solid arrows represent the optimal number of hops for the multihop DF relaying.

From these results, in a linear multihop network, the upper and lower bounds for K_{opt} are a function of R and α , but the exact K_{opt} is changed by other parameters of each system. Furthermore, the upper bound of K_{opt} can be obtained from a noise-limited environment using $f(K)$ with $\mathcal{I} = 0$, while the lower bound is determined by an interference-limited case using $f(K)$ with $\mathcal{N} = 0$. Intuitively, the reason that K_{opt} for interference-limited environment is the lower bound is because we consider the large scale path loss for randomly distributed interferers. Therefore, K_{opt} for multihop DF relaying in the presence of both noise and interference is bounded by it for noise-limited and interference-limited environments. This means that the exact K_{opt} increases or decreases within our bounds (11) in accordance with the relative effect of interference and noise.

IV. RESULTS AND DISCUSSION

To illustrate our analytical results, we adopt a Rayleigh fading channel with the fixed noise variance $N_0 = 1$ in the presence of both the correlated and independent interference. Fig. 2 plots the analytical outage probability as a function of the number of hops K for different λ and d_{SD} . From this figure, the spatial correlation in the interference reduces both K_{opt} and the outage performance. In addition, the performance gap between the independent and correlated interference increases in proportion to λ , K , and $1/d_{SD}$ due to growth the probability of located interferers at the same placement. Fig. 2 also reveals that K_{opt} for the multihop DF relaying decreases as the interference increases. Note that K_{opt} increases as d_{SD} grows, because multihop relaying is suited for transmission over long distances intuitively. To verify our upper and lower bounds, Fig. 3 presents K_{opt} for the multihop DF relaying in conjunction with noise/interference-limited environments. From this figure, we find that each bound is an exact solution for the noise-limited and interference-limited environment from analytical results and the exact K_{opt} is determined within our bounds. When $R = 0.5$ and $\alpha = 4$, the bounds of the independent interference $5 \leq K_{opt} \leq 8$ is more tight than the correlated case $1 \leq K_{opt} \leq 8$, because the correlated

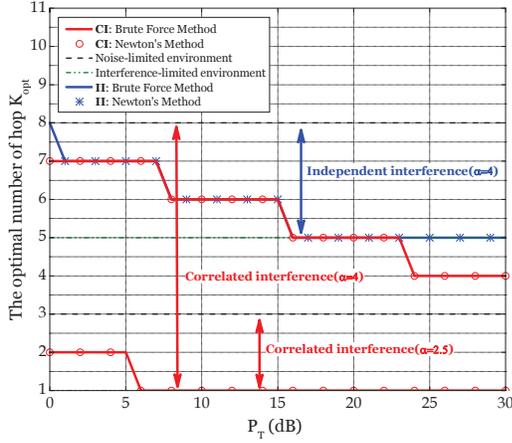


Fig. 3. The optimal number of hops for the multihop DF relaying with both correlated interference (CI) and independent interference (II) as a function of P_T for different $\alpha = 2.5$ and 4 with $d_{SD} = 3$, $\lambda = 0.01$, $R = 0.5$, and $P_{INF} = 5\text{dB}$.

interference decreases the lower bound for interference-limited environment. Furthermore, it is observed that K_{opt} decreases as the transmission power increases, because high transmission power allows transmission with smaller hops.

APPENDIX

A. Proof of the convexity for (4)

To verify convexity of our optimization problem (4), Hessian matrix $\mathbf{H}(f)$ should be positive definite, where $f(\mathbf{P}, \mathbf{d}, \beta) = \sum_{n=1}^K \frac{N_0 d_n^\alpha}{\mathcal{P}_n} \left(2^{\frac{R}{\beta_n}} - 1\right) + Q d_n^2 \left(\frac{2^{\frac{R}{\beta_n}} - 1}{\mathcal{P}_n}\right)^\delta$. As the $\mathbf{H}(f)$ is symmetric matrix, we prove that the determinant of every submatrix of $\mathbf{H}(f)$ is positive.¹ As the proof for positive of $\det[\mathbf{H}_1]$ and $\det[\mathbf{H}_2]$ can be obtained easily, we arrive at the desired result (4) by proving positive of $\det[\mathbf{H}_3]$ with the help of $\frac{2\beta + R \ln 2}{R \ln 2} > 1$.

B. Proof of Theorem 1

To compute the optimal resource allocation and placement of the relay nodes, we first define new variables as

$$\mathcal{A} \triangleq \frac{\mathcal{T} \left(2^{\frac{R}{\beta}} - 1\right)}{\mathcal{P}}, \quad \mathcal{B} \triangleq \frac{2^{\frac{R}{\beta_n}} - 1}{\mathcal{P}_n}, \quad \mathcal{C} \triangleq \frac{\mathcal{A}}{\mathcal{T}} \left(\frac{\mathcal{T} \mathcal{P}_n}{\mathcal{P}} - 1\right) \quad (16)$$

where $\mathcal{P} \triangleq \mathcal{P}_T - \sum_{n=1}^{K-1} \beta_n \mathcal{P}_n$ and $\mathcal{T} \triangleq 1 - \sum_{n=1}^{K-1} \beta_n$.

From (6), each $\frac{\partial f(\mathbf{P}, \mathbf{d}, \beta)}{\partial \mathcal{P}_n} = 0$, $\frac{\partial f(\mathbf{P}, \mathbf{d}, \beta)}{\partial d_n} = 0$, and $\frac{\partial f(\mathbf{P}, \mathbf{d}, \beta)}{\partial \beta_n}$

¹As a symmetric matrix A is positive definite if and only if the determinant of every submatrix of A is positive.

can be expressed in order as follows:

$$\frac{2Q\beta_n \mathcal{D}^2}{\alpha \mathcal{P}} \mathcal{A}^\delta + \frac{N_0 \beta_n}{\mathcal{P} \mathcal{D}^{-\alpha}} \mathcal{A} = \frac{2Q d_n^2}{\alpha \mathcal{P}_n} \mathcal{B}^\delta + \frac{N_0}{\mathcal{P}_n d_n^{-\alpha}} \mathcal{B}, \quad (17)$$

$$2Q \mathcal{D} \mathcal{A}^\delta + N_0 \alpha \mathcal{D}^{\alpha-1} \mathcal{A} = 2Q d_n \mathcal{B}^\delta + N_0 \alpha d_n^{\alpha-1} \mathcal{B}, \quad (18)$$

$$\begin{aligned} N_0 \left(\frac{\mathcal{A} \mathcal{D}^\alpha}{\mathcal{P} / \mathcal{P}_n} + \frac{2^{\frac{R}{\beta}} R \ln 2}{\mathcal{D}^{-\alpha} \mathcal{T} \mathcal{P}} \right) + \left(\mathcal{C} + \frac{2^{\frac{R}{\beta}} R \ln 2}{\mathcal{T} \mathcal{P}} \right) \frac{2Q \mathcal{A}^{\delta-1}}{\alpha \mathcal{D}^{-2}} \\ = N_0 \left(\frac{\mathcal{A} \mathcal{D}^\alpha}{\mathcal{T}} + \frac{2^{\frac{R}{\beta_n}} R \ln 2}{d_n^{-\alpha} \beta_n^2 \mathcal{P}_n} \right) + \left(\frac{2^{\frac{R}{\beta_n}} R \ln 2}{\beta_n^2 \mathcal{P}_n} \right) \frac{2Q \mathcal{B}^{\delta-1}}{\alpha d_n^{-2}} \end{aligned} \quad (19)$$

where $\mathcal{D} \triangleq d_{SD} - \sum_{n=1}^{K-1} d_n$. As the common solution of (17)–(19) is the same as (7), we complete the proof.

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