

On OFDM Ranging Accuracy in Multipath Channels

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Abstract—Wireless networks with ranging and localization capabilities enable new applications. The performance of wireless ranging depends on both the transmission technique and the propagation channel. Understanding such dependence is crucial to the design of high-accuracy ranging algorithms. In this paper, we derive the performance bounds for orthogonal frequency-division multiplexing (OFDM) ranging in multipath channels based on Fisher information analysis. In particular, we determine the equivalent Fisher information for ranging and quantify the effects of signal and channel parameters on the OFDM ranging accuracy. We also illustrate the ranging accuracy for different signal and channel parameters through numerical examples.

Index Terms—Cramér–Rao bound (CRB), equivalent Fisher information (EFI), multipath channel, orthogonal frequency-division multiplexing (OFDM), ranging.

I. INTRODUCTION

LOCATION-AWARENESS is essential to many commercial and military applications in future wireless networks [1], [2]. The Global Positioning System (GPS) is currently the most widely used localization system, but its performance is limited in indoor and urban areas. In such environments, location-awareness can be obtained via wireless localization networks [1]–[7], where the location of a node is estimated based on the waveforms received from reference nodes and prior knowledge.

The performance limits of wireless localization are determined in [8]–[13] using the Cramér–Rao bound (CRB) [14]. As shown in [8], ranging accuracy is one of the major factors affecting the localization performance [15]–[19]. Although a general framework has been developed for the fundamental limits of wireless localization in [8] and [9], further research is required to analyze the ranging and localization performance using specific transmission techniques.

Orthogonal frequency-division multiplexing (OFDM) is one of the most prominent wideband transmission techniques for

next-generation wireless communications [20]. The multiband OFDM (MB-OFDM) [21], [22], an ultra-wide bandwidth transmission scheme, has the potential to support both high data rates and high-accuracy ranging due to its wide bandwidth [23], [24]. Several OFDM ranging algorithms have been designed in previous works [25]–[31], and the CRB for OFDM ranging has been studied in [32]–[37]. In [32], the CRB is derived for single-path channels. In [33] and [34], the CRBs for single-path channels are derived by approximating each subcarrier as a narrow-band signal. In [35], the multipath components are assumed to be separable, and the corresponding CRB is essentially the same as that for single-path channels. In [36] and [37], the CRB is derived by assuming that all the channel parameters are known except for the propagation delay of the first path. To the best of the authors' knowledge, there is no existing work that quantifies the effects of signal parameters and path overlap in multipath channels on OFDM ranging accuracy.

In this paper, we determine the performance bounds for OFDM ranging in multipath channels based on Fisher information analysis. The main contributions of this paper are as follows.

- 1) We derive the Fisher information matrix (FIM) for multipath channel parameter estimation using OFDM signals and show that the FIM can be decomposed as a sum of those for different subcarriers.
- 2) We determine the equivalent Fisher information (EFI) for OFDM ranging in multipath channels as a function of the signal and channel parameters.
- 3) We quantify the effects of the signal and channel parameters on the CRB for OFDM ranging and characterize the gap between the CRBs for multipath and single-path channels.

The rest of this paper is organized as follows. Section II introduces the channel and OFDM signal models, Section III provides the expressions of the FIM and EFI for OFDM ranging, and Section IV analyzes the effects of the signal and multipath channel parameters on the EFI for OFDM ranging. Numerical and simulation results are provided in Section V, and conclusions are drawn in Section VI.

Notations: $[\cdot]^T$ denotes the transpose of its argument; c^* , $|c|$, and $\text{Re}\{c\}$ represent the conjugate, the modulus, and the real part of the complex number c , respectively; $\mathbb{E}_{\mathbf{x}}\{\cdot\}$ denotes the expectation of its argument with respect to the random vector \mathbf{x} ; $\mathbf{A} \succeq \mathbf{B}$ means that matrix $\mathbf{A} - \mathbf{B}$ is positive semidefinite; $[\cdot]_{i,j}$ denotes the element at the i th row and j th column of its argument; and $\text{diag}\{x_1, x_2, \dots, x_L\}$ is a diagonal matrix with x_1, x_2, \dots, x_L being its diagonal

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elements. Given $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_n]^T$ and $\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_n]^T$, the first- and second-order gradient operators are defined as $\nabla_{\mathbf{x}} \triangleq [\partial/\partial x_1 \ \partial/\partial x_2 \ \cdots \ \partial/\partial x_n]^T$ and $\Delta_{\mathbf{x}}^T \triangleq \nabla_{\mathbf{x}} \nabla_{\mathbf{y}}^T$.

II. SYSTEM MODEL

In this section, we first introduce the multipath channel model and the OFDM signal model. Then, we determine the sufficient statistic of the received signal for OFDM ranging.

A. Channel and Signal Models

We consider internode ranging using OFDM signals in multipath channels. The channel impulse response is

$$h(t) = \sum_{l=1}^L \alpha_l \delta(t - \tau_l)$$

where α_l and τ_l are the amplitude and delay of the l th path and L is the number of multipath components. Let $\boldsymbol{\theta} \triangleq [\tau_1 \ \alpha_1 \ \tau_2 \ \alpha_2 \ \cdots \ \tau_L \ \alpha_L]^T$ be the vector of the unknown channel parameters. Since the range between two nodes is the product of τ_1 and the propagation speed of the signal, estimating the range is equivalent to estimating τ_1 .¹

Let N be the number of subcarriers, $\mathcal{K} \triangleq \{0, 1, \dots, N-1\}$ be the set of subcarrier indices, A_k and ϕ_k be the amplitude and phase of the deterministic symbol modulated on the k th subcarrier, f_c be the central frequency, Δf be the subcarrier spacing, and $f_k = (2k - N + 1)\Delta f/2$ be the baseband frequency of the k th subcarrier. In particular, we consider that $f_c/\Delta f$ is an integer. Furthermore, let T_{CP} be the length of the cyclic prefix (CP) with $T_{\text{CP}} > \tau_L$, and define $g_0(t) \triangleq u(t + T_{\text{CP}}) - u(t - T)$, where $T = 1/\Delta f$ and $u(t)$ is the unit step function.

Then, the transmitted OFDM symbol is given by

$$s_0(t) = s(t)g_0(t)$$

where

$$s(t) = \sum_{k \in \mathcal{K}} A_k \cos(2\pi(f_c + f_k)t + \phi_k)$$

and the received signal is

$$r_0(t) = \int_0^{\tau_L} h(u)s_0(t-u)du + n(t)$$

in which $n(t)$ is the additive white Gaussian noise with the two-sided spectrum density of $N_0/2$. The receiver removes the CP and obtains

$$r(t) = r_0(t)g(t) = x(t) + n(t)g(t) \quad (1)$$

¹Here, we consider line-of-sight conditions and synchronized network, and τ_1 is the propagation delay of the signal along the direct path between the transmitter and the receiver. The analysis can be extended to non-line-of-sight conditions and asynchronous networks by adding parameters that characterize the resulting distance offset [8].

where $g(t) \triangleq u(t) - u(t - T)$ and

$$\begin{aligned} x(t) &= \left[\int_0^{\tau_L} h(u)s(t-u)g_0(t-u)du \right] \cdot g(t) \\ &= \left[\int_0^{\tau_L} h(u)s(t-u)du \right] \cdot g(t) \end{aligned}$$

in which the second equality is due to the fact that $g_0(t-u) = 1$ for $t \in [0, T]$ and $u \in [0, \tau_L]$ as $(t-u) \in [-\tau_L, T]$.

B. Sufficient Statistic for OFDM Ranging

The received signal (1) in the frequency domain is

$$R(f) = X(f) + N(f)$$

where $R(f)$ is the Fourier transform (FT) of $r(t)$, $N(f)$ is the FT of $n(t)g(t)$, and $X(f)$ is the FT of $x(t)$, given by

$$\begin{aligned} X(f) &= \sum_{k \in \mathcal{K}} \frac{A_k}{2} [H(f_c + f_k)G(f - f_c - f_k)e^{j\phi_k} \\ &\quad + H(-f_c - f_k)G(f + f_c + f_k)e^{-j\phi_k}] \quad (2) \end{aligned}$$

in which $H(f)$ and $G(f)$ are the FTs of $h(t)$ and $g(t)$, respectively, given by²

$$G(f) = T \text{sinc}(\pi f T) e^{-j\pi f T}$$

and

$$H(f) = \sum_{l=1}^L \alpha_l e^{-j2\pi f \tau_l}.$$

Let $\mathcal{R} \triangleq \{R(f) : f = i\Delta f, i \in \mathbb{Z}\}$ be a set of frequency-domain samples of $R(f)$ with Δf being the sample interval. By the sampling theorem [38], since $r(t)$ is time limited in $[0, T]$, $R(f)$ can be represented by \mathcal{R} .

Proposition 1: The noise components at different sampled frequencies $\{N(f) : f = i\Delta f, i \in \mathbb{Z}\}$ are independent identically distributed (i.i.d.) complex Gaussian random variables (RVs) with zero mean and variance $N_f = N_0 T/2$. □

Proof: See Appendix A. □

Let $\mathcal{R}_s \triangleq \{R(f) : f \in \mathcal{F}\}$ be a set of frequency-domain samples with $\mathcal{F} \triangleq \{\pm(f_c + f_k) : 0 \leq k \leq N-1\}$. We have the following result.

Proposition 2: The set \mathcal{R}_s is a sufficient statistic for estimating the channel parameters. Moreover

$$R(f) = \frac{TA_k}{2} \sum_{l=1}^L \alpha_l e^{-j(2\pi f \tau_l - \phi_k)} + N(f), \quad f \in \mathcal{F}. \quad (3)$$

Proof: Based on (2), for $f = i\Delta f$, $i \in \mathbb{Z}$, $X(f)$ is nonzero only if $f \in \mathcal{F}$, and $R(f)$ in (3) can be obtained after some algebra. Since the frequency-domain samples are independent of each other, it follows from the Theorem of Irrelevance [39] that the finite set \mathcal{R}_s is a sufficient statistic for estimating the channel parameters. □

²The function $\text{sinc}(t)$ is the unnormalized sinc function, defined by $\text{sinc}(t) = \sin(t)/t$, and $j = \sqrt{-1}$.

Remark 1: Proposition 2 shows that the range can be estimated based on the finite set \mathcal{R}_s without loss of information.

III. FIM FOR OFDM RANGING

In this section, we derive the FIM for estimating the multipath channel parameters based on the sufficient statistic \mathcal{R}_s . Both deterministic and random channel parameters are considered. For notational convenience, we denote $\omega = 2\pi f$, $\omega_c = 2\pi f_c$, $\Delta\omega = 2\pi\Delta f$, and $\omega_k = 2\pi f_k$ in the rest of this paper.

A. Deterministic Channel Parameters

First, we consider θ as deterministic and unknown and let $\hat{\theta} \triangleq [\hat{\tau}_1 \hat{\alpha}_1 \hat{\tau}_2 \hat{\alpha}_2 \cdots \hat{\tau}_L \hat{\alpha}_L]^T$ be an unbiased estimator of θ . The mean-squared estimation error (MSE) of $\hat{\theta}$ is bounded as

$$\mathbb{E}_{\mathcal{R}_s} \{(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T\} \succeq \mathbf{J}_\theta^{-1} \quad (4)$$

where \mathbf{J}_θ is the $2L \times 2L$ FIM, given by

$$\mathbf{J}_\theta = \mathbb{E}_{\mathcal{R}_s} \{-\Delta_\theta^2 \ln \Lambda(\mathcal{R}_s; \theta)\} \quad (5)$$

in which $\Lambda(\cdot; \theta)$ is the likelihood function. Since the noise components at different samples in \mathcal{R}_s are i.i.d. complex Gaussian by Proposition 1, $\Lambda(\cdot; \theta)$ can be expressed as the product of the likelihood functions at different subcarriers, i.e.,

$$\Lambda(\mathcal{R}_s; \theta) = \prod_{k \in \mathcal{K}} \Lambda(R(f_c + f_k); \theta) \Lambda(R(-f_c - f_k); \theta). \quad (6)$$

Using (3) and Proposition 1, we have

$$\Lambda(R(f); \theta) \propto \exp\left(-\frac{1}{N_f} \left| R(f) - \frac{TA_k}{2} \sum_{l=1}^L \alpha_l e^{-j(\omega\tau_l - \phi_k)} \right|^2\right)$$

for $f \in \mathcal{F}$.

Proposition 3: The FIM \mathbf{J}_θ can be decomposed as

$$\mathbf{J}_\theta = 2 \sum_{k \in \mathcal{K}} \mathbf{J}(f_c + f_k) \quad (7)$$

where

$$\mathbf{J}(f) = \mathbb{E}_{\mathcal{R}_s} \{-\Delta_\theta^2 \ln \Lambda(R(f); \theta)\}$$

is the FIM at frequency f with elements given by

$$\mathbb{E}_{\mathcal{R}_s} \{-\Delta_{\tau_i}^2 \ln \Lambda(R(f); \theta)\} = \frac{\alpha_i \alpha_j T}{N_0} P_k \omega^2 \cos(\omega\tau_{ij}) \quad (8)$$

$$\mathbb{E}_{\mathcal{R}_s} \{-\Delta_{\tau_i}^{\alpha_j} \ln \Lambda(R(f); \theta)\} = -\frac{\alpha_i T}{N_0} P_k \omega \sin(\omega\tau_{ij}) \quad (9)$$

and

$$\mathbb{E}_{\mathcal{R}_s} \{-\Delta_{\alpha_i}^{\alpha_j} \ln \Lambda(R(f); \theta)\} = \frac{T}{N_0} P_k \cos(\omega\tau_{ij}) \quad (10)$$

in which $f \in \mathcal{F}$, $\tau_{ij} = \tau_i - \tau_j$, $1 \leq i, j \leq L$, and $P_k = A_k^2$ is the power allocated on the k th subcarrier.

Proof: The result can be obtained from (5) and (6) after some algebra. Note also that $\mathbf{J}(f) = \mathbf{J}(-f)$ since its elements (8), (9), and (10) are even functions of f . \square

Remark 2: Proposition 3 shows that the FIM is a function of the channel parameters (i.e., multipath amplitudes $\{\alpha_i\}$ and path separations $\{\tau_{ij}\}$) and the OFDM signal parameters (i.e., carrier frequencies, the number of subcarriers, and the power allocation over the subcarriers). However, the FIM does not depend on the data symbol phases $\{\phi_k\}$, which carry no information about ranging.

In order to analyze the Fisher information for ranging, we consider the following partition of \mathbf{J}_θ :

$$\mathbf{J}_\theta = \begin{bmatrix} a & \mathbf{b}^T \\ \mathbf{b} & \mathbf{C} \end{bmatrix} \quad (11)$$

where $a \in \mathbb{R}$, $\mathbf{b} \in \mathbb{R}^{(2L-1) \times 1}$, and $\mathbf{C} \in \mathbb{R}^{(2L-1) \times (2L-1)}$.

Definition 1 (Equivalent Fisher Information): The equivalent fisher information (EFI) $J_e(\tau_1)$ for OFDM ranging is defined as

$$J_e(\tau_1) \triangleq a - \mathbf{b}^T \mathbf{C}^{-1} \mathbf{b}. \quad (12)$$

Remark 3: Let $\sigma_{\text{CRB}}^2(\tau_1)$ be the CRB for the MSE of $\hat{\tau}_1$. By (4) and (12), we have

$$\mathbb{E}_{\mathcal{R}_s} \{(\hat{\tau}_1 - \tau_1)^2\} \geq \sigma_{\text{CRB}}^2(\tau_1) = J_e^{-1}(\tau_1)$$

i.e., evaluating the CRB is equivalent to evaluating the EFI $J_e(\tau_1)$. Thus, the rest of this paper focuses on analyzing $J_e(\tau_1)$.

Proposition 4: The EFI for OFDM ranging in an L -path channel is given by

$$J_e(\tau_1) = \frac{2\alpha_1^2 T}{N_0} \left(\sum_{k \in \mathcal{K}} P_k \omega_{ck}^2 - \check{\mathbf{b}}^T \check{\mathbf{C}}^{-1} \check{\mathbf{b}} \right) \quad (13)$$

where $\omega_{ck} \triangleq \omega_c + \omega_k$, $\check{\mathbf{b}} = \sum_{k \in \mathcal{K}} P_k \mathbf{b}_k \mathbf{W}_k$, and $\check{\mathbf{C}} = \sum_{k \in \mathcal{K}} P_k \mathbf{W}_k \mathbf{C}_k \mathbf{W}_k$, in which \mathbf{b}_k and \mathbf{C}_k are given by (14) and

$$\mathbf{b}_k = \begin{bmatrix} 0 & \cos(\omega_{ck}\tau_{12}) & \sin(\omega_{ck}\tau_{12}) & \cdots & \cos(\omega_{ck}\tau_{1L}) & \sin(\omega_{ck}\tau_{1L}) \end{bmatrix}^T \quad (14)$$

$$\mathbf{C}_k = \begin{bmatrix} 1 & \sin(\omega_{ck}\tau_{12}) & \cos(\omega_{ck}\tau_{12}) & \cdots & \sin(\omega_{ck}\tau_{1L}) & \cos(\omega_{ck}\tau_{1L}) \\ \sin(\omega_{ck}\tau_{12}) & 1 & 0 & \cdots & \cos(\omega_{ck}\tau_{2L}) & -\sin(\omega_{ck}\tau_{2L}) \\ \cos(\omega_{ck}\tau_{12}) & 0 & 1 & \cdots & \sin(\omega_{ck}\tau_{2L}) & \cos(\omega_{ck}\tau_{2L}) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sin(\omega_{ck}\tau_{1L}) & \cos(\omega_{ck}\tau_{2L}) & \sin(\omega_{ck}\tau_{2L}) & \cdots & 1 & 0 \\ \cos(\omega_{ck}\tau_{1L}) & -\sin(\omega_{ck}\tau_{2L}) & \cos(\omega_{ck}\tau_{2L}) & \cdots & 0 & 1 \end{bmatrix} \quad (15)$$

(15) shown at the bottom of the previous page and $\mathbf{W}_k \in \mathbb{R}^{(2L-1) \times (2L-1)}$ is given by

$$\mathbf{W}_k = \text{diag}\{1, \omega_{ck}, 1, \omega_{ck}, 1, \dots, \omega_{ck}, 1\}.$$

Proof: The blocks in (11) can be written as

$$\begin{aligned} a &= \frac{2\alpha_1^2 T}{N_0} \sum_{k \in \mathcal{K}} P_k \omega_{ck}^2 \\ \mathbf{b}^T &= \frac{2\alpha_1 T}{N_0} \check{\mathbf{b}}^T \check{\mathbf{\Xi}} \\ \mathbf{C} &= \frac{2T}{N_0} \check{\mathbf{\Xi}} \check{\mathbf{C}} \check{\mathbf{\Xi}} \end{aligned} \quad (16)$$

where $\check{\mathbf{\Xi}} = \text{diag}\{1, \alpha_2, 1, \alpha_3, \dots, \alpha_L, 1\}$. Substituting the above equations into (12) leads to (13). \square

Remark 4: The term $\check{\mathbf{b}}^T \check{\mathbf{C}}^{-1} \check{\mathbf{b}}$ in (13) describes the effect of *path overlap*. Since \mathbf{J}_θ is positive semidefinite, \mathbf{C} is also positive semidefinite, and so is $\check{\mathbf{C}}$ by (16). Thus, $\check{\mathbf{b}}^T \check{\mathbf{C}}^{-1} \check{\mathbf{b}} \geq 0$ in (13), i.e., the EFI for OFDM ranging is reduced due to the path overlap in multipath channels. As a special case, the EFI in the single-path channel is given by

$$J_e(\tau_1) = [\mathbf{J}_\theta]_{1,1} = \frac{2\alpha_1^2 T}{N_0} \sum_{k \in \mathcal{K}} P_k \omega_{ck}^2. \quad (17)$$

In addition, (13) also shows that $J_e(\tau_1)$ does not depend on α_i for $i > 1$.

B. Random Channel Parameters

We now consider the channel parameters as RVs. With *a priori* knowledge of the channel parameters, the estimation error is lower bounded as

$$\mathbb{E}_{\mathcal{R}_s, \theta} \{(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T\} \succeq \bar{\mathbf{J}}_\theta^{-1}$$

where

$$\begin{aligned} \bar{\mathbf{J}}_\theta &= \mathbb{E}_{\mathcal{R}_s, \theta} \{-\Delta_\theta^\theta \ln \Lambda(\mathcal{R}_s, \theta)\} \\ &= \mathbb{E}_\theta \{-\Delta_\theta^\theta \ln p(\theta)\} + \mathbb{E}_\theta \{\mathbf{J}_\theta\} \end{aligned}$$

in which the first term is the contribution of the *a priori* knowledge of θ , with $p(\theta)$ being the probability density function of θ , and the second term is given by

$$\mathbb{E}_\theta \{\mathbf{J}_\theta\} = 2 \sum_{k \in \mathcal{K}} \mathbb{E}_\theta \{\mathbf{J}(f_c + f_k)\}.$$

Proposition 5: If the multipath delays $\boldsymbol{\tau} \triangleq [\tau_1 \ \tau_2 \ \dots \ \tau_L]^T$ are Poisson arrivals with rate λ and are independent of the amplitudes, then the elements of $\mathbb{E}_\theta \{\mathbf{J}(f)\}$ are

$$\begin{aligned} &\mathbb{E}_\theta \left\{ \mathbb{E}_{\mathcal{R}_s | \theta} \left[-\Delta_{\tau_i}^{\tau_j} \ln \Lambda(R(f) | \theta) \right] \right\} \\ &= \frac{\mathbb{E}\{\alpha_i \alpha_j\} T \omega^2 P_k \lambda^{|\mu_{ij}|} \cos(\mu_{ij} \arctan(\omega/\lambda))}{N_0 (\omega^2 + \lambda^2)^{|\mu_{ij}|/2}} \end{aligned} \quad (18)$$

$$\begin{aligned} &\mathbb{E}_\theta \left\{ \mathbb{E}_{\mathcal{R}_s | \theta} \left[-\Delta_{\tau_i}^{\alpha_j} \ln \Lambda(R(f) | \theta) \right] \right\} \\ &= -\frac{\mathbb{E}\{\alpha_i\} T \omega P_k \lambda^{|\mu_{ij}|} \sin(\mu_{ij} \arctan(\omega/\lambda))}{N_0 (\omega^2 + \lambda^2)^{|\mu_{ij}|/2}} \end{aligned} \quad (19)$$

and

$$\begin{aligned} &\mathbb{E}_\theta \left\{ \mathbb{E}_{\mathcal{R}_s | \theta} \left[-\Delta_{\tau_i}^{\alpha_j} \ln \Lambda(R(f) | \theta) \right] \right\} \\ &= \frac{T P_k \lambda^{|\mu_{ij}|} \cos(\mu_{ij} \arctan(\omega/\lambda))}{N_0 (\omega^2 + \lambda^2)^{|\mu_{ij}|/2}} \end{aligned} \quad (20)$$

where $\mu_{ij} = i - j$.

Proof: See Appendix B. \square

Remark 5: Note that $\bar{\mathbf{J}}_\theta$ is a function of the OFDM signal parameters and does not depend on instantaneous channel realizations.

IV. PERFORMANCE ANALYSIS OF OFDM RANGING

In this section, we quantify the effects of the carrier frequency, the power allocation, the number of subcarriers, and the subcarrier spacing on the EFI for OFDM ranging.

A. Single-Path Channel

We consider the following two types of constraints on the energy of an OFDM symbol. First, the total energy of the data part in the OFDM symbol is constrained as

$$\int_0^T s^2(t) dt = \frac{T}{2} \sum_{k \in \mathcal{K}} P_k \leq E_T \quad (21)$$

where E_T is the maximum total transmit energy. As shown in (17), the EFI for OFDM ranging in the single-path channel is the sum of the powers $\{P_k\}$ weighted by $\{\omega_{ck}^2\}$. Therefore, $J_e(\tau_1)$ in (17) can be maximized by allocating all the transmit power to the subcarrier with the highest frequency. In this case

$$J_e(\tau_1) = \frac{4\alpha_1^2 E_T}{N_0} (\omega_c + \omega_{N-1})^2. \quad (22)$$

Moreover, in addition to (21), we consider individual energy constraints, i.e., the energy of each subcarrier in an OFDM symbol is constrained as

$$\frac{T}{2} P_k \leq E_S, \quad k \in \mathcal{K}$$

where $E_S = E_T/N_S$ is the maximum transmit energy for each subcarrier, in which $N_S \in \mathbb{Z}$ and $0 < N_S \leq N$. According to (17), $J_e(\tau_1)$ can be maximized by allocating full energy to the N_S subcarriers with the highest frequencies, i.e.,

$$P_k = \frac{2E_S}{T}, \quad k = N - N_S, \dots, N - 1$$

which leads to

$$\begin{aligned} J_e(\tau_1) &= \frac{4\alpha_1^2 E_S}{N_0} \sum_{k=0}^{N_S-1} \left[\tilde{\omega}_c + \left(k - \frac{N_S-1}{2} \right) \Delta\omega \right]^2 \\ &= \frac{4\alpha_1^2 E_S N_S}{N_0} \left[\tilde{\omega}_c^2 + \frac{\Delta\omega^2}{12} (N_S^2 - 1) \right] \end{aligned} \quad (23)$$

where $\tilde{\omega}_c = \omega_c + (N - N_S)\Delta\omega/2$. Furthermore, based on (22) and (23), $J_e(\tau_1)$ increases quadratically with f_c and Δf under both the total and individual energy constraints.

B. Multipath Channel

As shown in (13), the closed-form expression of the EFI $J_e(\tau_1)$ for an L -path channel ($L > 1$) is difficult to obtain due to the matrix inversion in (13), which hinders the understanding about the effects of signal parameters and path overlap on the ranging accuracy. In order to gain insights into such effects, we mainly focus on two-path channels. Moreover, we approximate the elements in \mathbf{J}_θ except for $[\mathbf{J}_\theta]_{1,1}$ as

$$\sum_{k \in \mathcal{K}} P_k \omega_{ck}^2 \cos(\omega_{ck} \tau_{ij}) \approx \omega_c^2 \sum_{k \in \mathcal{K}} P_k \cos(\omega_{ck} \tau_{ij}) \quad (24)$$

$$\sum_{k \in \mathcal{K}} P_k \omega_{ck} \sin(\omega_{ck} \tau_{ij}) \approx \omega_c \sum_{k \in \mathcal{K}} P_k \sin(\omega_{ck} \tau_{ij}) \quad (25)$$

and

$$\sum_{k \in \mathcal{K}} P_k \omega_{ck}^2 \approx \omega_c^2 \sum_{k \in \mathcal{K}} P_k \quad (26)$$

considering that the central frequency of OFDM signals is usually much larger than the bandwidth.³ Note that we still have the exact EFI in the single-path channel (17) under these approximations since $[\mathbf{J}_\theta]_{1,1}$ is not approximated. In addition, since the EFI increases with the transmit energy, we rewrite the total energy constraint (21) with equality, i.e.,

$$\sum_{k \in \mathcal{K}} P_k = \frac{2E_T}{T}. \quad (27)$$

Proposition 6: Based on (24)–(27), the EFI for OFDM ranging in the two-path channel is given by

$$\tilde{J}_e(\tau_1) = \frac{2\alpha_1^2 T}{N_0} \left(\sum_{k \in \mathcal{K}} P_k \omega_{ck}^2 - \frac{\omega_c^2 T}{2E_T} h \right) \quad (28)$$

where

$$h = \left[\sum_{k \in \mathcal{K}} P_k \cos(\omega_k \tau) \right]^2 + \left[\sum_{k \in \mathcal{K}} P_k \sin(\omega_k \tau) \right]^2 \quad (29)$$

characterizes the effect of path overlap with $\tau = \tau_2 - \tau_1$.

Proof: See Appendix C. \square

Next, we quantify the effects of individual signal parameters on the ranging accuracy based on (28) under the total energy constraint (27).

1) *Central Frequency:* The EFI (28) can be rewritten as

$$\tilde{J}_e(\tau_1) = \frac{2\alpha_1^2 T}{N_0} \left[\left(\frac{2E_T}{T} - \frac{Th}{2E_T} \right) \omega_c^2 + \sum_{k \in \mathcal{K}} P_k (2\omega_c \omega_k + \omega_k^2) \right]$$

where $0 \leq h \leq 4E_T^2/T^2$ as implied by (29) and (35). Thus, for a given channel realization, $\tilde{J}_e(\tau_1)$ increases quadratically with f_c .

2) *Power Allocation:* The power allocation across the subcarriers can be optimized to maximize the EFI for OFDM

ranging, which is formulated as follows:

$$\begin{aligned} \mathcal{P}: \quad & \max_{\mathbf{P}} \quad \tilde{J}_e(\tau_1) \\ & \text{s.t.} \quad \sum_{k \in \mathcal{K}} P_k = \frac{2E_T}{T} \\ & \quad P_k \geq 0, \quad k \in \mathcal{K} \end{aligned} \quad (30)$$

where $\mathbf{P} \triangleq [P_0 \ P_1 \ \dots \ P_{N-1}]^T$.

Proposition 7: Problem \mathcal{P} is convex.

Proof: The objective function (30) can be written as

$$\tilde{J}_e(\tau_1) = \frac{2\alpha_1^2 T}{N_0} \left(-\frac{\omega_c^2 T}{2E_T} \mathbf{P}^T \mathbf{H}^T \mathbf{H} \mathbf{P} + \mathbf{w}^T \mathbf{P} \right) \quad (31)$$

where

$$\mathbf{H} = \begin{bmatrix} \cos(\omega_0 \tau) & \cos(\omega_1 \tau) & \dots & \cos(\omega_{N-1} \tau) \\ \sin(\omega_0 \tau) & \sin(\omega_1 \tau) & \dots & \sin(\omega_{N-1} \tau) \end{bmatrix}$$

and $\mathbf{w} = [\omega_{c0}^2 \ \omega_{c1}^2 \ \dots \ \omega_{cN-1}^2]^T$. Since $\mathbf{H}^T \mathbf{H}$ is positive semidefinite, (31) is a concave function of \mathbf{P} . Moreover, the constraints in (30) are linear functions of \mathbf{P} . Therefore, (30) is a convex optimization problem [40]. \square

Remark 6: Although the optimal power allocation depends on the unknown parameter τ , robust or adaptive power allocation algorithms can be designed based on an estimation of τ . The design of such algorithms is beyond the scope of this paper.

3) *Number of Subcarriers:* Here, we employ equal power allocation across the subcarriers, i.e., $P_k = 2E_T/(NT)$.

Proposition 8: Under equal power allocation, $\tilde{J}_e(\tau_1)$ can be expressed as

$$\tilde{J}_e(\tau_1) = \frac{4\alpha_1^2 E_T}{N_0} \left[\omega_c^2 + \frac{N^2 - 1}{12} \Delta\omega^2 - \frac{\omega_c^2}{N^2} \frac{\sin^2(N\Delta\omega\tau/2)}{\sin^2(\Delta\omega\tau/2)} \right]. \quad (32)$$

Proof: See Appendix D. \square

Since $\tilde{J}_e(\tau_1)$ is a function of τ , the effect of N on the EFI depends on instantaneous channel realizations. Alternatively, we analyze the effect of N on $\mathbb{E}_\tau\{\tilde{J}_e(\tau_1)\}$, where the multipath arrivals are modeled as a Poisson process with arrival rate λ .

Proposition 9: If τ follows exponential distribution with mean $1/\lambda$, then $\mathbb{E}_\tau\{\tilde{J}_e(\tau_1)\}$ is an increasing function of N , given by

$$\mathbb{E}_\tau\{\tilde{J}_e(\tau_1)\} = \frac{4\alpha_1^2 E_T}{N_0} \left[\omega_c^2 + \frac{N^2 - 1}{12} \Delta\omega^2 - \frac{\omega_c^2 \lambda^2}{\Delta\omega^2} q(N) \right] \quad (33)$$

where

$$q(N) = \frac{1}{N^2} \sum_{k \in \mathcal{K}} \sum_{l=0}^{N-1} \frac{1}{\lambda^2 / \Delta\omega^2 + (k-l)^2}. \quad (34)$$

Proof: See Appendix E. \square

Remark 7: Proposition 9 implies that the EFI can be increased by using more subcarriers. In particular, when N is large, $\mathbb{E}_\tau\{\tilde{J}_e(\tau_1)\}$ increases approximately quadratically with N .

³Simulation results in Section V show that the CRB based on the approximation (24)–(26) is accurate for MB-OFDM.

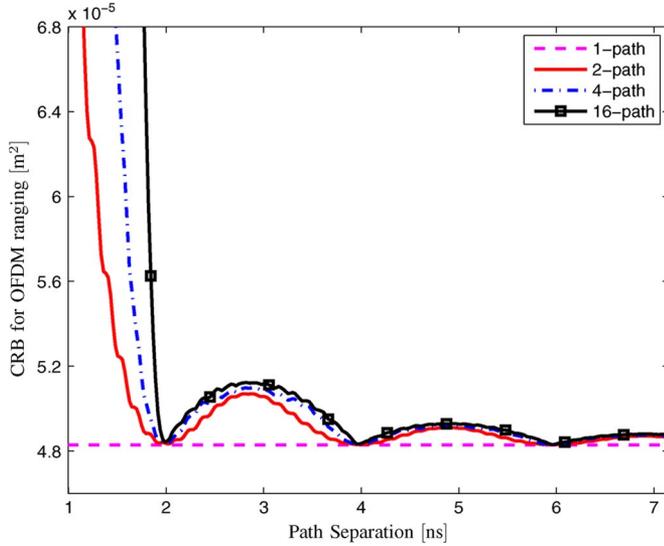


Fig. 1. CRB for OFDM ranging as a function of the path separation for different numbers of paths.

4) *Subcarrier Spacing*: Finally, we analyze the effect of Δf on the ranging performance. Again, we consider equal power allocation across the subcarriers. As shown in (33), $\mathbb{E}_{\tau}\{\tilde{J}_e(\tau_1)\}$ is an increasing function of Δf . When Δf is large, $\mathbb{E}_{\tau}\{\tilde{J}_e(\tau_1)\}$ increases approximately quadratically with Δf .

C. Discussion

The aforementioned analysis shows that the CRB in the single-path channel is not applicable for the design of OFDM ranging signals in multipath channels. For example, in order to minimize the CRB for OFDM ranging in single-path channels, we should allocate all the transmit power to the subcarrier with the highest frequency under the total energy constraint (27). However, if we apply the same power allocation for ranging in multipath channels, then the corresponding FIM is singular, i.e., the delays and amplitudes cannot be jointly estimated. Therefore, the CRB derived in this paper [e.g., the inverse of (13) and (28)] is a more accurate characterization of the OFDM ranging accuracy in multipath channels, yielding a basis for the effective design of OFDM signals for ranging.

V. NUMERICAL EXAMPLES

In this section, we evaluate the effects of signal and channel parameters on the CRB for OFDM ranging through numerical examples. We use the first operation band of MB-OFDM [21], [22] as the baseline signal model, which consists of 122 subcarriers with central frequency of 3.432 GHz and subcarrier spacing of 4.125 MHz. In addition, we consider total energy constraint (27).

Since the CRB for OFDM ranging does not depend on the multipath amplitudes α_i 's except for α_1 , we consider the signal-to-noise ratio (SNR) of the first path in the received signal, i.e.,

$$\text{SNR} = \frac{\alpha_1^2 \int_0^T s^2(t) dt}{N_0} = \frac{\alpha_1^2 E_T}{N_0}$$

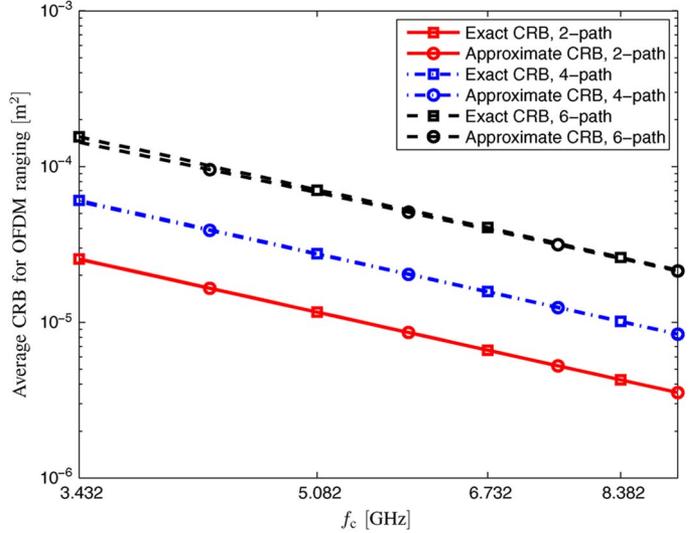


Fig. 2. Average CRB and its approximation as functions of the central frequency for different numbers of paths in log-scale.

and we choose E_T such that $\text{SNR} = 0$ dB in the following numerical examples.

A. Effects of Multipath Propagation

We first evaluate the path-overlap effect on the CRB for OFDM ranging. Here, we consider equally spaced multipath components and equal power allocation across the subcarriers. Figure 1 plots the CRB as a function of the path spacing. It shows that the CRB increases with the number of paths. Moreover, the CRB curves for multipath channels are close to that for the two-path channel, which implies that the effect of path overlap is dominated by the second path. Finally, the fluctuations in the CRB curves for multipath channels result from the fluctuations in the shape of the OFDM waveform. Note that previous works such as [32]–[35] only provide CRBs in single-path channels, i.e., the dashed line in Fig. 1. As shown in the figure, such bounds are too loose for evaluating the OFDM ranging accuracy in multipath channels.

B. Effects of OFDM Signal Parameters

Instead of evaluating the effects of signal parameters on $\sigma_{\text{CRB}}^2(\tau_1)$, which depends on instantaneous channel realizations, we illustrate the effects of f_c , N , and Δf on $\mathbb{E}_{\tau}\{\sigma_{\text{CRB}}^2(\tau_1)\}$ through Monte Carlo simulation, where the multipath arrivals are modeled as a Poisson process with arrival rate λ .⁴ Moreover, we evaluate the effect of power allocation $\{P_k\}$ on $\sigma_{\text{CRB}}^2(\tau_1)$ in two-path channels with path separation being deterministic. In the following results, we compare the approximate CRBs using (24)–(26) with the exact CRBs and show that the analysis in Section IV based on these approximations is accurate.

1) *Central Frequency*: In Fig. 2, $\mathbb{E}_{\tau}\{\sigma_{\text{CRB}}^2(\tau_1)\}$ is plotted as a function of f_c in log-scale with equal power allocation across the subcarriers and $\lambda = 1 \text{ ns}^{-1}$. As shown in Fig. 2, the

⁴We exclude path separations smaller than 0.7 ns in the simulations.

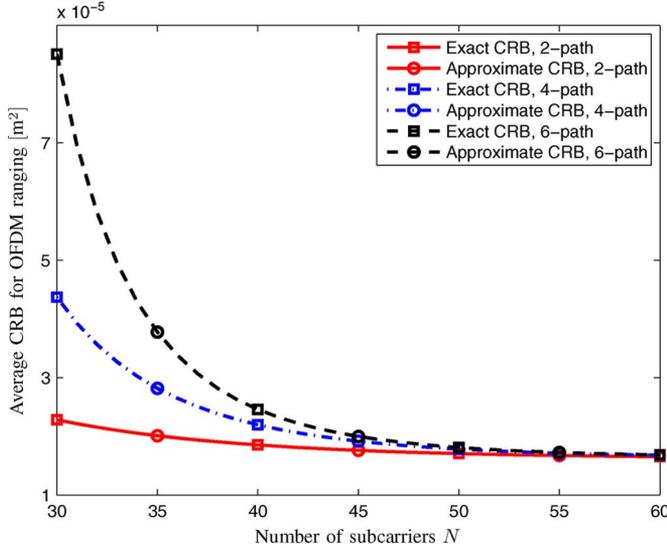


Fig. 3. Average CRB and its approximation as functions of the number of subcarriers for different numbers of paths.

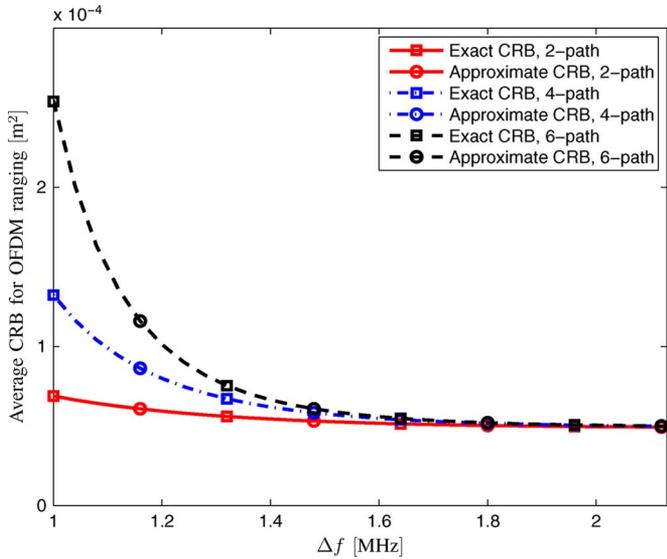


Fig. 4. Average CRB and its approximation as functions of the subcarrier spacing for different numbers of paths.

CRBs in different multipath channels decrease at the same rate with f_c . This result agrees with the analysis in Section IV-B1, i.e., the CRBs for OFDM ranging decrease quadratically with f_c for multipath channels.

2) *Number of Subcarriers*: Figure 3 shows the effect of N on $\mathbb{E}_{\tau}\{\sigma_{\text{CRB}}^2(\tau_1)\}$ with equal power allocation across the subcarriers and $\lambda = 0.25 \text{ ns}^{-1}$. As shown in the figure, the CRB for OFDM ranging decreases as N increases, which agrees with the analysis in Section IV-B3. In addition, the benefit of increasing the number of subcarriers diminishes as N increases, and the reduction of the CRB with N is more significant for channels with larger numbers of multipath components.

3) *Subcarrier Spacing*: In Fig. 4, $\mathbb{E}_{\tau}\{\sigma_{\text{CRB}}^2(\tau_1)\}$ is plotted as a function of Δf with equal power allocation across the subcarriers and $\lambda = 0.25 \text{ ns}^{-1}$. The CRB for OFDM ranging is shown to decrease with Δf , which agrees with the analysis in

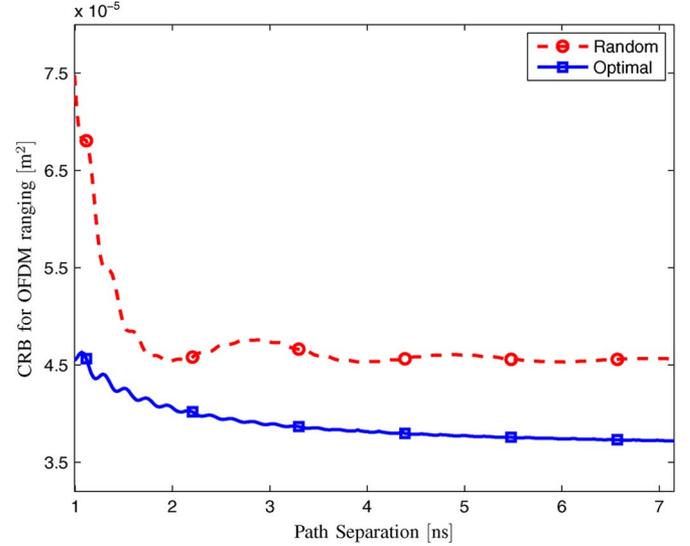


Fig. 5. CRBs with optimal and random power allocations as a function of the path separation.

Section IV-B4. As shown in the figure, the reduction of the CRB diminishes as Δf increases, and the benefit of increasing Δf is more significant for channels with larger numbers of multipath components.

4) *Power Allocation*: Figure 5 compares the CRBs for OFDM ranging using optimal and random power allocations in two-path channels. The dashed curve is the averaged CRB over 10^4 randomly generated power allocations. The figure shows that the benefit of optimal power allocation is more significant for small path separations.

VI. CONCLUSION

In this paper, we derived the performance bounds for OFDM ranging based on Fisher information analysis. In particular, we decomposed the FIM for parameter estimation of multipath channels into the sum of those FIMs for different subcarriers. Based on the EFI, we analyzed the effects of signal and channel parameters on the OFDM ranging accuracy. Our results provide the quantitative relationship between the ranging accuracy and the central frequency, the subcarrier spacing, the number of subcarriers, and the power allocation across the subcarriers. These results can serve as guidelines for designing OFDM ranging systems.

APPENDIX A PROOF OF PROPOSITION 1

The FT of the noise can be written as

$$N(f) = \int_{-\infty}^{\infty} n(t)g(t)e^{-j2\pi ft} dt = N_{\text{R}}(f) + jN_{\text{I}}(f)$$

where

$$N_{\text{R}}(f) = \int_{-\infty}^{\infty} n(t)g(t) \cos(2\pi ft) dt$$

$$N_{\text{I}}(f) = \int_{-\infty}^{\infty} n(t)g(t) \sin(2\pi ft) dt.$$

Note that $\{g(t) \cos(2\pi ft), g(t) \sin(2\pi ft)\}$ are \mathcal{L}_2 functions, i.e., square integrable, and the inner products of the white Gaussian process $n(t)$ with \mathcal{L}_2 functions are jointly Gaussian RVs [38]. Thus, $\{N_R(f), N_I(f) : f = i\Delta f, i \in \mathbb{Z}\}$ are jointly Gaussian. Moreover, for two sampled frequencies f_1 and f_2

$$\begin{aligned} \mathbb{E}\{N_R(f_1)N_R(f_2)\} &= \mathbb{E}\left\{\int_0^T \frac{N_0}{2} \cos(2\pi f_1 t) \cos(2\pi f_2 t) dt\right\} \\ &= \begin{cases} \frac{N_0 T}{4}, & \text{if } f_1 = f_2, \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

where we use the fact that $f_1 + f_2 = K_1 \Delta f$ and $f_1 - f_2 = K_2 \Delta f$, $K_1, K_2 \in \mathbb{Z}$. Similarly, we have $\mathbb{E}\{N_I(f_1)N_I(f_2)\} = N_0 T/4$ only if $f_1 = f_2$, and $\mathbb{E}\{N_R(f_1)N_I(f_2)\} = 0$. Thus, $\{N(f) : f = i\Delta f, i \in \mathbb{Z}\}$ are i.i.d. complex Gaussian RVs, and we have

$$\mathbb{E}\{N(f_1)N^*(f_2)\} = \begin{cases} \frac{N_0 T}{2}, & \text{if } f_1 = f_2, \\ 0, & \text{otherwise.} \end{cases}$$

APPENDIX B PROOF OF PROPOSITION 5

Let $\boldsymbol{\alpha} \triangleq [\alpha_1 \alpha_2 \cdots \alpha_L]^T$. Since $\boldsymbol{\tau}$ and $\boldsymbol{\alpha}$ are independent, we have

$$\mathbb{E}_{\boldsymbol{\theta}}\{\mathbf{J}(f)\} = \mathbb{E}_{\boldsymbol{\alpha}}\{\mathbb{E}_{\boldsymbol{\tau}}\{\mathbf{J}(f)\}\}$$

and the elements of $\mathbf{J}(f)$ are given in (8)–(10). For Poisson arrivals, $|\tau_{ij}|$ ($i \neq j$) is modeled as an Erlang variable with probability density function

$$f_{|\tau_{ij}|}(t) = \frac{\lambda^{|\mu_{ij}|} t^{|\mu_{ij}|-1} e^{-\lambda t}}{(|\mu_{ij}|-1)!}, \quad t \geq 0$$

where $\mu_{ij} = i - j$. According to [41], we have

$$\mathbb{E}_{|\tau_{ij}|}\{\cos(2\pi f \tau_{ij})\} = \frac{\lambda^{|\mu_{ij}|} \cos(\mu_{ij} \arctan(2\pi f / \lambda))}{((2\pi f)^2 + \lambda^2)^{|\mu_{ij}|/2}}$$

and

$$\mathbb{E}_{|\tau_{ij}|}\{\sin(2\pi f \tau_{ij})\} = \frac{\lambda^{|\mu_{ij}|} \sin(\mu_{ij} \arctan(2\pi f / \lambda))}{((2\pi f)^2 + \lambda^2)^{|\mu_{ij}|/2}}.$$

Thus, the elements in $\mathbb{E}_{\boldsymbol{\theta}}\{\mathbf{J}(f)\}$ are given by (18), (19), and (20). These equations are also applicable for $i = j$.

APPENDIX C PROOF OF PROPOSITION 6

Based on (24)–(26), we obtain the approximate FIM for the two-path channel as follows:

$$\tilde{\mathbf{J}}_e = \begin{bmatrix} \mathbf{A} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{D} \end{bmatrix}$$

where

$$\mathbf{A} = \frac{2T}{N_0} \sum_{k \in \mathcal{K}} P_k \begin{bmatrix} \alpha_1^2 \omega_{ck}^2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{B} = \frac{2T}{N_0} \sum_{k \in \mathcal{K}} P_k \begin{bmatrix} \alpha_1 \alpha_2 \omega_c^2 \cos(\omega_{ck} \tau_{12}) & \alpha_2 \omega_c \sin(\omega_{ck} \tau_{12}) \\ -\alpha_1 \omega_c \sin(\omega_{ck} \tau_{12}) & \cos(\omega_{ck} \tau_{12}) \end{bmatrix}$$

$$\mathbf{D} = \frac{2T}{N_0} \begin{bmatrix} \frac{2\alpha_2^2 \omega_c^2 E_T}{T} & 0 \\ 0 & \frac{2E_T}{T} \end{bmatrix}.$$

We first calculate the EFI matrix $\tilde{\mathbf{J}}_e(\{\tau_1, \alpha_1\})$ for $\{\tau_1, \alpha_1\}$ as

$$\tilde{\mathbf{J}}_e(\{\tau_1, \alpha_1\}) = \mathbf{A} - \mathbf{B}^T \mathbf{D}^{-1} \mathbf{B} = \begin{bmatrix} J_1 & J_2 \\ J_2 & J_3 \end{bmatrix}.$$

Then, the EFI $\tilde{J}_e(\tau_1)$ for OFDM ranging is calculated as

$$\tilde{J}_e(\tau_1) = J_1 - J_2^2 / J_3.$$

After some algebra, we obtain

$$\tilde{J}_e(\tau_1) = \frac{2\alpha_1^2 T}{N_0} \left(\sum_{k \in \mathcal{K}} P_k \omega_{ck}^2 - \frac{\omega_c^2 T}{2E_T} h \right)$$

where

$$\begin{aligned} h &\triangleq \left[\sum_{k \in \mathcal{K}} P_k \cos(\omega_{ck} \tau) \right]^2 + \left[\sum_{k \in \mathcal{K}} P_k \sin(\omega_{ck} \tau) \right]^2 \\ &= \sum_{k \in \mathcal{K}} \sum_{l \in \mathcal{K}} P_k P_l \cos((\omega_k - \omega_l) \tau) \end{aligned} \quad (35)$$

which does not depend on ω_c . Thus, we obtain (28).

APPENDIX D PROOF OF PROPOSITION 8

With $P_k = 2E_T / (NT)$, h in (29) is

$$\begin{aligned} h &= \frac{4E_T^2}{N^2 T^2} \left\{ \left[\sum_{k \in \mathcal{K}} \cos(\omega_k \tau) \right]^2 + \left[\sum_{k \in \mathcal{K}} \sin(\omega_k \tau) \right]^2 \right\} \\ &= \frac{4E_T^2}{N^2 T^2} \left[\sum_{k \in \mathcal{K}} \cos(\omega_k \tau) \right]^2. \end{aligned}$$

Thus, the EFI in (28) becomes

$$\tilde{J}_e(\tau_1) = \frac{4\alpha_1^2 E_T}{N_0} \left\{ \frac{1}{N} \sum_{k \in \mathcal{K}} \omega_{ck}^2 - \frac{\omega_c^2}{N^2} \left[\sum_{k \in \mathcal{K}} \cos(\omega_k \tau) \right]^2 \right\} \quad (36)$$

where the first term in the brackets can be calculated as

$$\frac{1}{N} \sum_{k \in \mathcal{K}} (\omega_c + \omega_k)^2 = \omega_c^2 + \frac{N^2 - 1}{12} \Delta \omega^2$$

after some algebra and the second term is

$$\begin{aligned} \frac{\omega_c^2}{N^2} \left[\sum_{k \in \mathcal{K}} \cos(\omega_k \tau) \right]^2 &= \frac{\omega_c^2}{N^2} \left[\operatorname{Re} \left\{ \sum_{k \in \mathcal{K}} e^{j\omega_k \tau} \right\} \right]^2 \\ &= \frac{\omega_c^2}{N^2} \frac{\sin^2(M\Delta\omega\tau)}{\sin^2(\Delta\omega\tau/2)}. \end{aligned}$$

This leads to (32).

APPENDIX E PROOF OF PROPOSITION 9

The second term in the brackets of (36) depends on τ , where

$$\begin{aligned} \left[\sum_{k \in \mathcal{K}} \cos(\omega_k \tau) \right]^2 &= \left[\sum_{k \in \mathcal{K}} \cos(\omega_k \tau) \right]^2 + \left[\sum_{k \in \mathcal{K}} \sin(\omega_k \tau) \right]^2 \\ &= \sum_{k \in \mathcal{K}} \sum_{l \in \mathcal{K}} \cos((k-l)\Delta\omega\tau). \end{aligned}$$

After some algebra, the expectation of $\cos((k-l)\Delta\omega\tau)$ is

$$\mathbb{E}_\tau \{ \cos((k-l)\Delta\omega\tau) \} = \frac{\lambda^2}{\lambda^2 + (k-l)^2 \Delta\omega^2}$$

which leads to (33).

Since the second term in the brackets of (33) is an increasing function of N , it is sufficient to show that $q(N)$ in (34) is also an increasing function of N . Note that

$$(N+1)^2 [q(N) - q(N+1)] = \frac{N+1}{N\xi} + q_1(N) - 2q_2(N)$$

where

$$\begin{aligned} q_1(N) &= \frac{2N+1}{N^2} \sum_{k \in \mathcal{K}} \sum_{l \neq k} \frac{1}{\xi + (k-l)^2} \\ &\geq \frac{2N+1}{N} \left[q_2(N) - \frac{1}{\xi + N^2} \right] \end{aligned} \quad (37)$$

$$q_2(N) = \sum_{k \in \mathcal{K}} \frac{1}{\xi + (k-N)^2} \geq \frac{N}{\xi + N^2} \quad (38)$$

and $\xi = \lambda^2/\Delta\omega^2$. Therefore

$$\begin{aligned} (N+1)^2 [q(N) - q(N+1)] &\stackrel{(37)}{\geq} \frac{N+1}{N\xi} + \frac{1}{N} q_2(N) - \frac{2N+1}{N(\xi + N^2)} \\ &\stackrel{(38)}{\geq} \frac{N+1}{N\xi} - \frac{N+1}{N(\xi + N^2)} \\ &\geq 0 \end{aligned}$$

i.e., $q(N)$ decreases as N increases. Thus, $\mathbb{E}_\tau \{ \tilde{J}_e(\tau_1) \}$ is an increasing function of N .

REFERENCES

- [1] M. Z. Win, A. Conti, S. Mazuelas, Y. Shen, W. M. Gifford, D. Dardari, and M. Chiani, "Network localization and navigation via cooperation," *IEEE Commun. Mag.*, vol. 49, no. 5, pp. 56–62, May 2011.
- [2] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location: Challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 24–40, Jul. 2005.
- [3] H. Wymeersch, J. Lien, and M. Z. Win, "Cooperative localization in wireless networks," *Proc. IEEE*, vol. 97, no. 2, pp. 427–450, Feb. 2009.
- [4] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero III, R. L. Moses, and N. S. Correal, "Locating the nodes: Cooperative localization in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 54–69, Jul. 2005.
- [5] A. Conti, M. Guerra, D. Dardari, N. Decarli, and M. Z. Win, "Network experimentation for cooperative localization," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 2, pp. 467–475, Feb. 2012.
- [6] U. A. Khan, S. Kar, and J. M. F. Moura, "Distributed sensor localization in random environments using minimal number of anchor nodes," *IEEE Trans. Signal Process.*, vol. 57, no. 5, pp. 2000–2016, May 2009.
- [7] U. A. Khan, S. Kar, and J. M. F. Moura, "DILAND: An algorithm for distributed sensor localization with noisy distance measurements," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1940–1947, Mar. 2010.
- [8] Y. Shen and M. Z. Win, "Fundamental limits of wideband localization—Part I: A general framework," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4956–4980, Oct. 2010.
- [9] Y. Shen, H. Wymeersch, and M. Z. Win, "Fundamental limits of wideband localization—Part II: Cooperative networks," *IEEE Trans. Inf. Theory*, vol. 56, no. 10, pp. 4981–5000, Oct. 2010.
- [10] L. Mailaender, "On the geolocation bounds for round-trip time-of-arrival and all non-line-of-sight channels," *EURASIP J. Adv. Signal Process.*, vol. 2008, pp. 1–10, 2008.
- [11] Y. Shen and M. Z. Win, "On the accuracy of localization systems using wideband antenna arrays," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 270–280, Jan. 2010.
- [12] D. B. Jourdan, D. Dardari, and M. Z. Win, "Position error bound for UWB localization in dense cluttered environments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 2, pp. 613–628, Apr. 2008.
- [13] Y. Shen, S. Mazuelas, and M. Z. Win, "Network navigation: Theory and interpretation," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 9, pp. 1823–1834, Oct. 2012.
- [14] H. L. Van Trees, *Detection, Estimation and Modulation Theory*, vol. 1. New York, NY, USA: Wiley, 1968.
- [15] D. Dardari, A. Conti, U. J. Ferner, A. Giorgetti, and M. Z. Win, "Ranging with ultrawide bandwidth signals in multipath environments," *Proc. IEEE*, vol. 97, no. 2, pp. 404–426, Feb. 2009.
- [16] X. Wang, Z. Wang, and B. O'Dea, "A TOA-based location algorithm reducing the errors due to non-line-of-sight (NLOS) propagation," *IEEE Trans. Veh. Technol.*, vol. 52, no. 1, pp. 112–116, Jan. 2003.
- [17] H. Wymeersch, S. Maranó, W. M. Gifford, and M. Z. Win, "A machine learning approach to ranging error mitigation for UWB localization," *IEEE Trans. Commun.*, vol. 60, no. 6, pp. 1719–1728, Jun. 2012.
- [18] Y.-T. Chan, W.-Y. Tsui, H.-C. So, and P.-C. Ching, "Time of arrival based localization under NLOS conditions," *IEEE Trans. Veh. Technol.*, vol. 55, no. 1, pp. 17–24, Jan. 2006.
- [19] S. Maranó, W. M. Gifford, H. Wymeersch, and M. Z. Win, "NLOS identification and mitigation for localization based on UWB experimental data," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 7, pp. 1026–1035, Sep. 2010.
- [20] J. Chuang and N. Sollenberger, "Beyond 3G: Wideband wireless data access based on OFDM and dynamic packet assignment," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 78–87, Jul. 2000.
- [21] A. Batra, "Multi-band OFDM Physical Layer Proposal for IEEE 802.15 Task Group 3a," IEEE Working Group, Piscataway, NJ, USA, IEEE P802.15-03/268r3, Mar. 2004.
- [22] A. Batra, J. Balakrishnan, G. R. Aiello, J. R. Foerster, and A. Dabak, "Design of a multiband OFDM system for realistic UWB channel environments," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 9, pp. 2123–2138, Sep. 2004.
- [23] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios: A look at positioning aspects for future sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 70–84, Jul. 2005.
- [24] Y. Ohhikata and T. Kobayashi, "Proposal for an MB-OFDM UWB system simultaneously undertaking ranging and communications," in *Proc. IEEE Int. Conf. Ultra-Wideband*, Zurich, Switzerland, Sep. 2005, pp. 604–608.

- [25] X. Li and K. Pahlavan, "Super-resolution TOA estimation with diversity for indoor geolocation," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 224–234, Jan. 2004.
- [26] Z. J. Wang, Z. Han, and K. J. R. Liu, "A MIMO-OFDM channel estimation approach using time of arrivals," *IEEE Trans. Wireless Commun.*, vol. 4, no. 3, pp. 1207–1213, May 2005.
- [27] F. Zhao, W. Yao, C. C. Logothetis, and Y. Song, "Super-resolution TOA estimation in OFDM systems for indoor environments," in *Proc. IEEE Int. Conf. on Netw., Sens. and Control*, London, U.K., Apr. 2007, pp. 723–728.
- [28] H. Xu, C.-C. Chong, I. Guvenc, F. Watanabe, and L. Yang, "High-resolution TOA estimation with multi-band OFDM UWB signals," in *Proc. IEEE Int. Conf. Commun.*, Beijing, China, May 2008, pp. 4191–4196.
- [29] E. Saberinia and A. H. Tewfik, "Ranging in multiband ultrawideband communication systems," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2523–2530, Jul. 2008.
- [30] H. Xu, L. Yang, Y. T. J. Morton, and M. M. Miller, "Mistiming performance analysis of the energy detection based ToA estimator for MB-OFDM," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3980–3984, Aug. 2009.
- [31] F. Kocak, H. Celebi, S. Gezici, K. A. Qaraqe, H. Arslan, and H. V. Poor, "Time-delay estimation in dispersed spectrum cognitive radio systems," *EURASIP J. Adv. Signal Process.*, vol. 2010, pp. 1–10, Jan. 2010.
- [32] D. Wang and M. Fattouche, "OFDM transmission for time-based range estimation," *IEEE Signal Process. Lett.*, vol. 17, no. 6, pp. 571–574, Jun. 2010.
- [33] S. Gezici, H. Celebi, H. V. Poor, and H. Arslan, "Fundamental limits on time delay estimation in dispersed spectrum cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 78–83, Jan. 2009.
- [34] D. Dardari, Y. Karisan, S. Gezici, A. A. D'Amico, and U. Mengali, "Performance limits on ranging with cognitive radio," in *Proc. IEEE Int. Conf. Commun. Workshops*, Dresden, Germany, Jun. 2009, pp. 1–5.
- [35] D. Wang, M. Fattouche, and F. Ghannouchi, "Fundamental limit of OFDM range estimation in a separable multipath environment," *Circuits, Syst., Signal Process.*, vol. 31, no. 3, pp. 1215–1227, Jun. 2012.
- [36] R. Cardinali, L. D. Nardis, P. Lombardoand, and M.-G. D. Benedetto, "Lower bounds for ranging accuracy with multi band OFDM and direct sequence UWB signals," in *Proc. IEEE Int. Conf. Ultra-Wideband*, Zurich, Switzerland, Sep. 2005, pp. 302–307.
- [37] R. Cardinali, L. D. Nardis, M.-G. D. Benedetto, and P. Lombardo, "UWB ranging accuracy in high- and low-data-rate applications," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 4, pp. 1865–1875, Jun. 2006.
- [38] R. G. Gallager, *Principles of Digital Communication*. Cambridge, U.K.: Cambridge Univ. Press, 2008.
- [39] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*, 1st ed. London, U.K.: Wiley, 1965.
- [40] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
- [41] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. San Diego, CA, USA: Academic, 2007.



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