Asymptotic Statistics of Mutual Information for Doubly Correlated MIMO Channels

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***Abstract*— In this paper, we derive the asymptotic statistics of mutual information for multiple-input multiple-output (MIMO) Rayleigh-fading channels in the presence of spatial fading correlation at both the transmitter and the receiver. We first introduce a class of asymptotic linear spectral statistics, called *correlants*, for a structured correlation matrix. The mean and variance of MIMO mutual information are then expressed in terms of the correlants of spatial correlation matrices in the asymptotic regime where the number of transmit and receive antennas tends to infinity. In particular, using Szegö's theorem on the asymptotic eigenvalue distribution of Toeplitz matrices, we give examples for special classes of correlation matrices with Toeplitz structure—*exponential* (or *Kac–Murdock–Szegö*), *tridiagonal*, and *constant* (or *intraclass*) correlation matrices.**

***Index Terms*— Asymptotic linear spectral statistics, channel capacity, multiple-input multiple-output (MIMO) system, mutual information, Rayleigh fading, spatial fading correlation, Toeplitz form.**

I. Introduction

ABOUT a decade after the first assertion of [1] that the capacity generally grows proportionally with the number of antennas, the seminal work of [2] and [3] has explicitly shown that the capacity of a multiple-input multiple-output (MIMO) channel increases linearly with the minimum of the numbers of transmit and receive antennas in rich scattering propagation environments. Numerous prior investigations regarding the capacity or mutual information of MIMO systems are in two directions: exact or bound analysis for a finite number of antennas [2]–[7] and asymptotic analysis for a large number of antennas [3], [8]–[12]. Although the asymptotic analysis gives only an approximation for the case of finite antenna numbers, it often allows us to circumvent the difficult exact analysis and to provide more lucid insights into the capacity behavior.

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The essential mathematical tool used in understanding asymptotic capacity behavior is the theory of large dimensional random matrices (see, e.g., [13]–[17]). The crucial result in random matrix theory states that for a large class of random matrix ensembles, the empirical distribution of eigenvalues converges almost surely to a deterministic limiting distribution as the matrix dimension increases [13], [14]. Moreover, the central limit theorem argument for *linear spectral statistics (LSS)*[1] of large dimensional random matrices [15, Theorem 1.1] (or the central limit theorem for random determinants [16], [17]) reveals the interesting feature of MIMO capacity (as a random variable)—the "Gaussian behavior" in the limit as the number of transmit and receive antennas goes to infinity with a certain limiting ratio.

In [3], the exact and asymptotic mean (or ergodic) capacity for independent and identically distributed (i.i.d.) MIMO channels was derived in an integral form using the well-known random matrix results: the eigenvalue distribution of Wishart matrices [18] and the so-called *Marčenko–Pastur law* [13].[2] By directly applying [15, Theorem 1.1], the distribution of the capacity for i.i.d. MIMO channels was shown in [8] to have a Gaussian limit where the asymptotic variance was further derived. Also, Gaussianity of the capacity for i.i.d. MIMO channels was shown in [9] for various asymptotic scenarios in the number of antennas and the signal-to-noise ratio (SNR).

To characterize MIMO mutual information under more realistic propagation environments, several authors have dealt with the exact or asymptotic analysis in the presence of spatial fading correlation. Using the distribution theory of random matrices, the determinantal formulas for the exact characteristic function (CF) of mutual information were derived for "one-sided correlated" [5] and "doubly correlated (often called *Kronecker-correlated*)" [6] MIMO channels. Furthermore, using the determinantal CF formula, all statistical moments, including the skewness and kurtosis as well as the relevant first two moments (mean and variance), of the MIMO mutual information were obtained as trace formulas in [6].

It was shown in [10] that the ergordic mutual information for doubly correlated MIMO channels grows linearly with the number of antennas under some assumptions as in the i.i.d. case, but the asymptotic growth rate is smaller in the presence

[1]For any $n \times n$ positive semidefinite matrix $\mathbf{A}$, the LSS of $\mathbf{A}$ are quantities of the form

$$\frac{1}{n}\sum_{\ell=1}^{n} f\left(\lambda_\ell\left(\mathbf{A}\right)\right)$$

where $\lambda_\ell\left(\mathbf{A}\right)$, $\ell = 1, 2, \ldots, n$, denote the eigenvalues of $\mathbf{A}$ in any order and f is a function on $[0, \infty)$ [15].

[2]The closed-form expression for the exact mean capacity was presented in [4, Theorem III.1].

of spatial correlation. The analytical machinery used in [10] relies on the Stieltjes transform of the asymptotic eigenvalue distribution [14]. In parallel to this methodology, a so-called *replica* method has been used in [11] to derive the statistics of mutual information for large (but *finite*) numbers of antennas under both i.i.d. and doubly correlated fading.[3] More recently, asymptotic Gaussianity of the mutual information was shown in [12] for (exponentially) doubly correlated fading in a unidimensional limit where the number of antennas at either the transmitter or the receiver tends to infinity while the other side is fixed to have the finite number of antennas.

As a supplement to our recent exact analysis for the finite number of antennas [6], this paper deals with an asymptotic analysis for mutual information statistics of doubly correlated MIMO channels in the limit as n_{T} transmit and n_{R} receive antennas tend to infinity with a ratio $n_{\mathrm{T}}/n_{\mathrm{R}} \rightarrow \kappa$. For the doubly correlated case, it is difficult to apply the methodology of the Stieltjes transform (which has been successfully employed for i.i.d. or one-sided correlated MIMO channels) for the asymptotic mutual information analysis (in particular, beyond the first-order statistic). Instead, we begin with the results from the replica and saddle-point techniques in [11] for large (but finite) numbers of antennas. We then perform the asymptotic analysis for infinite antenna numbers (rather than a large antenna approximation), which allows us to simply characterize the mutual information statistics in terms of the ratio $n_{\mathrm{T}}/n_{\mathrm{R}}$ and the SNR at which MIMO systems operate—without an explicit value of n_{T} or n_{R}. The main contributions of the paper can be summarized as follows:

- We first introduce a class of asymptotic LSS for structured correlation matrices, referred to as the *correlants* (see Definition 2), to bridge the gap between the analytical machinery of *large* and *infinite* dimensional random matrix problems. Then, the asymptotic mean of mutual information per receive antenna, μ_{asy}, and the asymptotic variance, σ_{asy}^2, are expressed in terms of the correlants for arbitrary transmit and receive correlation matrices (see Proposition 1).
- Using Szegö's theorem on the asymptotic eigenvalue distribution of Toeplitz matrices [19]–[21], we present three examples of μ_{asy} and σ_{asy}^2 for specific classes of correlation matrices of Toeplitz form—*exponential* (or *Kac–Murdock–Szegö*), *tridiagonal*, and *constant* (or *intraclass*) correlation matrices (see Theorems 1–3).[4]
- We show that the asymptotic statistics μ_{asy} and σ_{asy}^2 as a function of the ratio $\kappa \geq 0$ are *monotonically increasing* and *unimodal*, respectively. From these monotone and unimodal properties, we determine the *maximum scaling* (with the number of receive antennas) and the *peak variation* of the mutual information (with respect to κ) for a given SNR in the asymptotic regime.

This paper is organized as follows. In Section II, the asymptotic mean and variance of the mutual information for doubly correlated MIMO channels are given in terms of the correlants of matrices with arbitrary correlation structure. Examples for special classes of Toeplitz correlation matrices are presented in Section III. In Section IV, some numerical results and discussion are provided to illustrate our asymptotic analysis. Finally, Section V concludes the paper.

II. Asymptotic Analysis

Let $\mathbf{H} \in \mathbb{C}^{n_{\mathrm{R}} \times n_{\mathrm{T}}}$ be a random channel matrix whose (i,j)th entries H_{ij}, $i = 1,2,\ldots,n_{\mathrm{R}}$, $j = 1,2,\ldots,n_{\mathrm{T}}$, are complex propagation coefficients between the jth transmit and the ith receive antennas with $\mathbb{E}\left\{|H_{ij}|^2\right\} = 1$.[5] We consider a doubly correlated Rayleigh-fading channel with the channel matrix $\mathbf{H}$ given by [10]

$$\mathbf{H} = \mathbf{\Psi}_{\mathrm{R}}^{1/2}\mathbf{H}_0\mathbf{\Psi}_{\mathrm{T}}^{1/2} \tag{1}$$

where $\mathbf{H}_0$ is the $n_{\mathrm{R}} \times n_{\mathrm{T}}$ random matrix with i.i.d., zero-mean, unit-variance, complex Gaussian entries and $\mathbf{\Psi}_{\mathrm{T}}$ and $\mathbf{\Psi}_{\mathrm{R}}$ are $n_{\mathrm{T}} \times n_{\mathrm{T}}$ transmit and $n_{\mathrm{R}} \times n_{\mathrm{R}}$ receive correlation matrices, respectively.[6] Note that $\mathbf{\Psi}_{\mathrm{T}}$ and $\mathbf{\Psi}_{\mathrm{R}}$ are non-random deterministic matrices.

When the receiver has perfect channel knowledge and the transmitter is constrained in its total average power with equal-power allocation to each of transmit antennas, the mutual information $\mathcal{I}$ between the input and output signals is given as a random variable [1]–[3]

$$\mathcal{I} = \ln\det\left(\mathbf{I}_{n_{\mathrm{R}}} + \frac{\bar{\gamma}}{n_{\mathrm{T}}}\mathbf{H}\mathbf{H}^{\dagger}\right) \qquad \text{nats/s/Hz} \tag{2}$$

where $\dagger$ denotes the transpose conjugate of a matrix, $\mathbf{I}_n$ is the $n \times n$ identity matrix, and $\bar{\gamma}$ is the average SNR at each receive antenna.

To characterize the asymptotic statistical behavior of $\mathcal{I}$ in the presence of spatial correlation, it is necessary to select a class of matrices with specific correlation structure. For generality, we begin by defining a class of asymptotic LSS for matrices with arbitrary correlation structure, which will be invoked in deriving asymptotic mutual information statistics.

A. Asymptotic LSS of Correlation Matrices: Correlants

Definition 1 (Structure of Correlation Matrices): Let $\mathbf{\Psi}_n$ be an $n \times n$ correlation matrix whose (i,j)th entry is determined by solely a function $\psi(i,j)$. Then, a class of correlation matrices $\{\mathbf{\Psi}_n\}_{n=1}^{\infty}$ is said to have the correlation structure ψ and $\mathbf{\Psi}_n$ is denoted by $\mathbf{\Psi}_n(\psi)$.

For example, for the uncorrelated case where all off-diagonal elements are equal to zero ($\mathbf{\Psi}_n = \mathbf{I}_n$), the correlation structure, denoted by ψ_δ, corresponds to $\psi_\delta(i,j) = \delta_{ij}$.[7]

Definition 2 (Correlant): Let $\mathbf{\Psi}_n(\psi)$ be an $n \times n$ correlation matrix with structure ψ. Then, the ℓth *correlant* $\Upsilon_\ell(\xi;\psi)$ of $\mathbf{\Psi}_n(\psi)$ with respect to a nonnegative quantity ξ is defined

[3]The replica method was introduced in statistical physics and has been used to analyze the spectral properties of a variety of random matrices (see, e.g., [14] and references therein).

[4]These correlation models have been widely used for many multiple-antenna communication problems (see, e.g., [4]–[6], [11], [12], [22]).

[5]Throughout the paper, $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ denote the natural numbers and the fields of real and complex numbers, respectively.

[6]A correlation matrix is a class of positive-semidefinite matrices with all diagonal entries 1.

[7]δ_{ij} denotes the Kronecker delta.

as an asymptotic LSS of the matrix with correlation structure ψ as

$$\Upsilon_\ell(\xi;\psi) \triangleq \lim_{n\to\infty} \frac{\tau_\ell}{n}\frac{d^\ell}{d\xi^\ell} \ln\det\Big\{\mathbf{I}_n + \xi\mathbf{\Psi}_n(\psi)\Big\} \tag{3}$$

where

$$\tau_\ell = \begin{cases} 1, & \text{if } \ell = 0 \\ \frac{(-1)^{\ell-1}}{(\ell-1)!}, & \text{otherwise.} \end{cases} \tag{4}$$

Let

$$F^{\mathbf{\Psi}_n(\psi)}(\lambda) \triangleq \frac{1}{n}\{\text{number of eigenvalues of } \mathbf{\Psi}_n(\psi) \le \lambda\} \tag{5}$$

be the *empirical eigenvalue distribution (EED)* of $\mathbf{\Psi}_n(\psi)$, which reflects the fraction of the eigenvalues less than or equal to λ.[8] If there exists an asymptotic eigenvalue distribution F^ψ to which the EED $F^{\mathbf{\Psi}_n(\psi)}$ converges in distribution as $n\to\infty$, then the correlant $\Upsilon_\ell(\xi;\psi)$ is given by

$$\Upsilon_\ell(\xi;\psi) = \int_{\lambda_{\min}(\psi)}^{\lambda_{\max}(\psi)} \mathcal{K}_\ell(\lambda,\xi)\, dF^\psi(\lambda) \tag{6}$$

with the kernel

$$\mathcal{K}_\ell(\lambda,\xi) \triangleq \begin{cases} \ln(1+\xi\lambda), & \text{if } \ell = 0 \\ \left(\frac{\lambda}{1+\xi\lambda}\right)^\ell, & \text{otherwise} \end{cases} \tag{7}$$

where

$$\lambda_{\min}(\psi) = \lim_{n\to\infty}\min_k \lambda_k(\mathbf{\Psi}_n(\psi)) \tag{8}$$

$$\lambda_{\max}(\psi) = \lim_{n\to\infty}\max_k \lambda_k(\mathbf{\Psi}_n(\psi)). \tag{9}$$

For example, the identity matrix $\mathbf{I}_n = \mathbf{\Psi}_n(\psi_\delta)$ has the correlant

$$\Upsilon_\ell(\xi;\psi_\delta) = \mathcal{K}_\ell(1,\xi). \tag{10}$$

B. Asymptotic Statistics of $\mathcal{I}$

Proposition 1 (Doubly Correlated MIMO Channel): Let $\mathbf{\Psi}_\mathrm{T}$ and $\mathbf{\Psi}_\mathrm{R}$ be of structure ψ_T and ψ_R such that there exist the correlants $\Upsilon_\ell(\xi;\psi_\mathrm{T})$ and $\Upsilon_\ell(\xi;\psi_\mathrm{R})$ at least up to the second order. Then, as n_T and n_R tend to infinity with a limiting ratio $n_\mathrm{T}/n_\mathrm{R}\to\kappa\ge 0$, the asymptotic mean of mutual information in nats/s/Hz per receive antenna is

$$\begin{aligned}\mu_\text{asy}(\bar\gamma,\kappa) &\triangleq \lim_{\substack{n_\mathrm{T},n_\mathrm{R}\to\infty\\ n_\mathrm{T}/n_\mathrm{R}\to\kappa}} \frac{1}{n_\mathrm{R}}\mathbb{E}\{\mathcal{I}\} \\ &= \kappa\Upsilon_0(\zeta_2;\psi_\mathrm{T}) + \Upsilon_0(\bar\gamma\zeta_1;\psi_\mathrm{R}) - \kappa\zeta_1\zeta_2 \end{aligned} \tag{11}$$

and the asymptotic variance is

$$\begin{aligned}\sigma^2_\text{asy}(\bar\gamma,\kappa) &\triangleq \lim_{\substack{n_\mathrm{T},n_\mathrm{R}\to\infty\\ n_\mathrm{T}/n_\mathrm{R}\to\kappa}} \mathbb{V}\mathrm{ar}\{\mathcal{I}\} \\ &= -\ln\left\{1 - \frac{\bar\gamma^2}{\kappa}\cdot\Upsilon_2(\zeta_2;\psi_\mathrm{T})\,\Upsilon_2(\bar\gamma\zeta_1;\psi_\mathrm{R})\right\} \end{aligned} \tag{12}$$

where the quantities $\zeta_k \ge 0$, $k = 1,2$, are determined by solving the system of equations

$$\zeta_1 = \Upsilon_1(\zeta_2;\psi_\mathrm{T}) \tag{13}$$

$$\zeta_2 = \frac{\bar\gamma}{\kappa}\cdot\Upsilon_1(\bar\gamma\zeta_1;\psi_\mathrm{R}). \tag{14}$$

This proposition can be obtained directly by using the replica and saddle-point analyses in [11] and taking the limit $n_\mathrm{T}, n_\mathrm{R}\to\infty$ with a ratio $n_\mathrm{T}/n_\mathrm{R}\to\kappa$, along with Definition 2. Note that if $\psi_\mathrm{T} = \psi_\mathrm{R} = \psi_\delta$ (i.e., i.i.d. MIMO channel), then it is easy to show that (11) and (12) in Proposition 1 reduce to the previously known results (see, e.g., [8]). As mentioned, the central limit theorem argument [15, Theorem 1.1] leads us to the asymptotic Gaussianity of mutual information with equal-power allocation for i.i.d. MIMO channels. This is also true for one-sided correlation at either the transmitter or the receiver (i.e., $\psi_\mathrm{T} = \psi_\delta$ or $\psi_\mathrm{R} = \psi_\delta$), because the case of $\psi_\mathrm{T} = \psi_\delta$ or $\psi_\mathrm{R} = \psi_\delta$ meets the conditions for [15, Theorem 1.1].

III. Examples: Correlation Matrices of Toeplitz Form

In this section, we give examples of asymptotic statistics of MIMO mutual information for correlation matrices of Toeplitz form. In particular, the correlation structures for $\mathbf{\Psi}_\mathrm{T}$ and $\mathbf{\Psi}_\mathrm{R}$ of interest are exponential, tridiagonal, and constant correlation. They are special classes of symmetric Toeplitz matrices and often appear in the multiple-antenna communication literature.

A. Exponential Correlation (Kac–Murdock–Szegö) Matrix

The nth-order positive-definite exponential correlation matrix with a correlation coefficient $\rho\in[0,1)$, denoted by $\mathbf{\Psi}_n\left(\psi_\rho^{[\mathrm{e}]}\right)$, is an $n\times n$ symmetric Toeplitz matrix of the following structure

$$\mathbf{\Psi}_n\left(\psi_\rho^{[\mathrm{e}]}\right) = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{bmatrix}_{n\times n} \tag{15}$$

which is sometimes called the nth-order Kac–Murdock–Szegö matrix. This model can be applicable to an equally-spaced linear array and the positive definiteness of $\mathbf{\Psi}_n\left(\psi_\rho^{[\mathrm{e}]}\right)$ holds for any $\rho\in[0,1)$ (see [23, Sec. 7.2, Prob. 12]).

Lemma 1: The EED of $\mathbf{\Psi}_n\left(\psi_\rho^{[\mathrm{e}]}\right)$ converges in distribution

[8] The EED of a random matrix is a random function because of the randomness of the eigenvalues themselves [13]–[15]. In contrast, (5) is a deterministic function for given structure ψ and dimension n.

to

$$\frac{dF^{\psi_\rho^{[e]}}(\lambda)}{d\lambda} = \begin{cases} \frac{1}{\pi\lambda\sqrt{\left(\lambda-\frac{1-\rho}{1+\rho}\right)\left(\frac{1+\rho}{1-\rho}-\lambda\right)}}, & \frac{1-\rho}{1+\rho} < \lambda < \frac{1+\rho}{1-\rho} \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

as $n \to \infty$.

Proof: See Appendix A. □

Theorem 1 (Exponential Correlation): If $\psi_\mathrm{T} = \psi_{\rho_\mathrm{T}}^{[e]}$ and $\psi_\mathrm{R} = \psi_{\rho_\mathrm{R}}^{[e]}$, then

$$\mu_\mathrm{asy}(\bar{\gamma},\kappa) = 2\kappa \ln\left\{ \frac{\sqrt{\frac{1-\rho_\mathrm{T}}{1+\rho_\mathrm{T}} + \zeta_2^{[e]}} + \sqrt{\frac{1+\rho_\mathrm{T}}{1-\rho_\mathrm{T}} + \zeta_2^{[e]}}}{\sqrt{\frac{1-\rho_\mathrm{T}}{1+\rho_\mathrm{T}}} + \sqrt{\frac{1+\rho_\mathrm{T}}{1-\rho_\mathrm{T}}}} \right\} + 2\ln\left\{ \frac{\sqrt{\frac{1-\rho_\mathrm{R}}{1+\rho_\mathrm{R}} + \bar{\gamma}\zeta_1^{[e]}} + \sqrt{\frac{1+\rho_\mathrm{R}}{1-\rho_\mathrm{R}} + \bar{\gamma}\zeta_1^{[e]}}}{\sqrt{\frac{1-\rho_\mathrm{R}}{1+\rho_\mathrm{R}}} + \sqrt{\frac{1+\rho_\mathrm{R}}{1-\rho_\mathrm{R}}}} \right\} - \kappa\zeta_1^{[e]}\zeta_2^{[e]} \tag{17}$$

and

$$\sigma^2_\mathrm{asy}(\bar{\gamma},\kappa) = -\ln\left\{ 1 - \frac{\bar{\gamma}^2}{4\kappa} \frac{\frac{1-\rho_\mathrm{T}}{1+\rho_\mathrm{T}} + \frac{1+\rho_\mathrm{T}}{1-\rho_\mathrm{T}} + 2\zeta_2^{[e]}}{\left[\left(\frac{1-\rho_\mathrm{T}}{1+\rho_\mathrm{T}} + \zeta_2^{[e]}\right)\left(\frac{1+\rho_\mathrm{T}}{1-\rho_\mathrm{T}} + \zeta_2^{[e]}\right)\right]^{3/2}} \times \frac{\frac{1-\rho_\mathrm{R}}{1+\rho_\mathrm{R}} + \frac{1+\rho_\mathrm{R}}{1-\rho_\mathrm{R}} + 2\bar{\gamma}\zeta_1^{[e]}}{\left[\left(\frac{1-\rho_\mathrm{R}}{1+\rho_\mathrm{R}} + \bar{\gamma}\zeta_1^{[e]}\right)\left(\frac{1+\rho_\mathrm{R}}{1-\rho_\mathrm{R}} + \bar{\gamma}\zeta_1^{[e]}\right)\right]^{3/2}} \right\} \tag{18}$$

where the quantities $\zeta_k^{[e]} \geq 0$, $k = 1, 2$, are determined by solving the system of equations

$$\zeta_1^{[e]} = \frac{1}{\sqrt{\left(\frac{1-\rho_\mathrm{T}}{1+\rho_\mathrm{T}} + \zeta_2^{[e]}\right)\left(\frac{1+\rho_\mathrm{T}}{1-\rho_\mathrm{T}} + \zeta_2^{[e]}\right)}} \tag{19}$$

$$\zeta_2^{[e]} = \frac{\bar{\gamma}/\kappa}{\sqrt{\left(\frac{1-\rho_\mathrm{R}}{1+\rho_\mathrm{R}} + \bar{\gamma}\zeta_1^{[e]}\right)\left(\frac{1+\rho_\mathrm{R}}{1-\rho_\mathrm{R}} + \bar{\gamma}\zeta_1^{[e]}\right)}}. \tag{20}$$

Proof: See Appendix B. □

B. Tridiagonal Correlation Matrix

The nth-order positive-definite tridiagonal correlation matrix with a correlation coefficient $\rho \in [0, 0.5)$, denoted by $\mathbf{\Psi}_n\left(\psi_\rho^{[t]}\right)$, is an $n \times n$ symmetric Toeplitz matrix of the following structure

$$\mathbf{\Psi}_n\left(\psi_\rho^{[t]}\right) = \begin{bmatrix} 1 & \rho & & & 0 \\ \rho & 1 & \rho & & \\ & \ddots & \ddots & \ddots & \\ & & \rho & 1 & \rho \\ 0 & & & \rho & 1 \end{bmatrix}_{n\times n}. \tag{21}$$

This model can be used for the benign-case analysis where only the adjacent signals are correlated. Since $\mathbf{\Psi}_n\left(\psi_\rho^{[t]}\right)$ is positive definite for $\rho \in \left[0, 0.5/\cos\frac{\pi}{n+1}\right)$ [22], the positive definiteness of $\mathbf{\Psi}_n\left(\psi_\rho^{[t]}\right)$ with $\rho \in [0, 0.5)$ holds for any positive integer n.

Lemma 2: The EED of $\mathbf{\Psi}_n\left(\psi_\rho^{[t]}\right)$ converges in distribution to

$$\frac{dF^{\psi_\rho^{[t]}}(\lambda)}{d\lambda} = \begin{cases} \frac{1}{\pi\sqrt{(\lambda-1+2\rho)(1+2\rho-\lambda)}}, & 1-2\rho < \lambda < 1+2\rho \\ 0, & \text{otherwise,} \end{cases} \tag{22}$$

as $n \to \infty$.

Proof: See Appendix A. □

Theorem 2 (Tridiagonal Correlation): If $\psi_\mathrm{T} = \psi_{\rho_\mathrm{T}}^{[t]}$ and $\psi_\mathrm{R} = \psi_{\rho_\mathrm{R}}^{[t]}$, then we have (23) and (24), shown at the bottom of the next page.

Proof: See Appendix C. □

C. Constant (or Intraclass) Correlation Matrix

The nth-order positive-definite constant correlation matrix with a correlation coefficient $\rho \in [0, 1)$, denoted by $\mathbf{\Psi}_n\left(\psi_\rho^{[c]}\right)$, is an $n \times n$ symmetric Toeplitz matrix of the following structure

$$\mathbf{\Psi}_n\left(\psi_\rho^{[c]}\right) = \begin{bmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{bmatrix}_{n\times n}. \tag{27}$$

This model can be used for the worst-case analysis or for rough approximations using the average value of correlation coefficients for all off-diagonal entries of the correlation matrix. Since the eigenvalues of $\mathbf{\Psi}_n\left(\psi_\rho^{[c]}\right)$ are $1-\rho$ with multiplicity $n-1$ and $1+(n-1)\rho$, the positive definiteness of $\mathbf{\Psi}_n\left(\psi_\rho^{[c]}\right)$ holds for any $\rho \in [0, 1)$.

Theorem 3 (Constant Correlation): If $\psi_\mathrm{T} = \psi_{\rho_\mathrm{T}}^{[c]}$ and $\psi_\mathrm{R} = \psi_{\rho_\mathrm{R}}^{[c]}$, then

$$\mu_\mathrm{asy}(\bar{\gamma},\kappa) = \kappa\ln\left\{1 + \frac{\varrho_\mathrm{c}\bar{\gamma}}{\kappa} - \varrho_\mathrm{c}\bar{\gamma}\mathcal{B}(\bar{\gamma},\varrho_\mathrm{c},\kappa)\right\} + \ln\left\{1 + \varrho_\mathrm{c}\bar{\gamma} - \varrho_\mathrm{c}\bar{\gamma}\mathcal{B}(\bar{\gamma},\varrho_\mathrm{c},\kappa)\right\} - \kappa\mathcal{B}(\bar{\gamma},\varrho_\mathrm{c},\kappa) \tag{28}$$

and

$$\sigma^2_\mathrm{asy}(\bar{\gamma},\kappa) = -\ln\left\{1 - \kappa\mathcal{B}^2(\bar{\gamma},\varrho_\mathrm{c},\kappa)\right\} \tag{29}$$

where $\varrho_\mathrm{c} = (1-\rho_\mathrm{T})(1-\rho_\mathrm{R}) \in (0, 1]$ and

$$\mathcal{B}(\bar{\gamma},\varrho,\kappa) = \frac{1}{2}\left(1 + \frac{1}{\kappa} + \frac{1}{\varrho\bar{\gamma}} - \sqrt{\left(1 + \frac{1}{\kappa} + \frac{1}{\varrho\bar{\gamma}}\right)^2 - \frac{4}{\kappa}}\right). \tag{30}$$

Proof: See Appendix D. □

Comparing Theorem 3 with the i.i.d. results reveals that the effect of constant correlation on the asymptotic behavior is to reduce the SNR by the factor ϱ_c. We also remark that the sequence $\left\{c_k^{[c]}\right\}_{k=-\infty}^{\infty}$ with

$$c_k^{[c]} = \begin{cases} 1, & \text{if } k = 0 \\ \rho, & \text{otherwise} \end{cases} \tag{31}$$

is not absolutely summable for $\rho \in (0,1)$. Therefore, the asymptotic theory of Toeplitz matrices cannot be applied to constant correlation and the asymptotic EED $F^{\psi_\rho^{[c]}}$ is not feasible. However, we can find the asymptotic statistics by means of the correlants of $\mathbf{\Psi}_n\left(\psi_\rho^{[c]}\right)$.

IV. Numerical Results and Discussion

In this section, we give some numerical results and discussion on our asymptotic analysis.

A. Verification

Figs. 1–6 verify the results in Theorems 1–3.[9] Figs. 1 and 2 show the simulated mean (nats/s/Hz) per receive antenna and variance of $\mathcal{I}$ as a function of n_R. These curves are compared with their asymptotic values for doubly correlated MIMO channels with exponential correlation $\mathbf{\Psi}_\mathrm{T} = \mathbf{\Psi}_{n_\mathrm{T}}\left(\psi_{\rho_\mathrm{T}}^{[e]}\right)$ and $\mathbf{\Psi}_\mathrm{R} = \mathbf{\Psi}_{n_\mathrm{R}}\left(\psi_{\rho_\mathrm{R}}^{[e]}\right)$. In these figures, $n_\mathrm{T} = n_\mathrm{R}$ (i.e., $\kappa = 1$), $\rho_\mathrm{T} = \rho_\mathrm{R} = 0$ (i.i.d.), 0.5, 0.8, and $\bar{\gamma} = 15$ dB. It can be seen that as the number of antennas grows, the simulated results converge rapidly to their asymptotic values given by Theorem 1. Similar observations are made for tridiagonal correlation in Figs. 3 and 4 where $\mathbf{\Psi}_\mathrm{T} = \mathbf{\Psi}_{n_\mathrm{T}}\left(\psi_{0.3}^{[t]}\right)$, $\mathbf{\Psi}_\mathrm{R} = \mathbf{\Psi}_{n_\mathrm{R}}\left(\psi_{0.3}^{[t]}\right)$, and $\bar{\gamma} = 15$ dB when $n_\mathrm{T} = n$, $n_\mathrm{R} = 2n$ ($\kappa = 0.5$); $n_\mathrm{T} = n_\mathrm{R} = n$ ($\kappa = 1$); and $n_\mathrm{T} = 2n$, $n_\mathrm{R} = n$ ($\kappa = 2$). The corresponding asymptotic values are obtained using Theorem 2.

In Figs. 5 and 6, the simulated results and their asymptotic values given by Theorem 3 are depicted for constant correlation in the same environment as in Figs. 1 and 2. It can be observed that the convergence rates for both the mean and the variance are much slower than those for exponential and tridiagonal correlation. This slow convergence is due to the fact that the largest eigenvalue $\lambda_{\text{max}} = 1 + (n-1)\rho$ of $\mathbf{\Psi}_n\left(\psi_\rho^{[c]}\right)$ grows with n and hence, its contribution (even with unit multiplicity) in LSS vanishes slowly as n increases—i.e., the asymptotic contribution amounting to $\lim_{n\to\infty} \frac{1}{n}\ln\left(1+\xi\lambda_{\text{max}}\right)$ in the correlants becomes zero owing to unit multiplicity irrespective of n, but it is of $O\left(\frac{\ln n}{n}\right)$ rather than $O\left(n^{-1}\right)$ due to the increase in λ_{max} with n.

B. Asymptotic Maximum Growth Rate

Fig. 7 shows the asymptotic mean $\mu_{\text{asy}}\left(\bar{\gamma},\kappa\right)$ in nats/s/Hz per receive antenna as a function of the ratio κ at $\bar{\gamma} = 15$ dB for exponential correlation of the following six cases: i) $\rho_\mathrm{T} = \rho_\mathrm{R} = 0$; ii) $\rho_\mathrm{T} = 0.5$, $\rho_\mathrm{R} = 0$; iii) $\rho_\mathrm{T} = 0.9$, $\rho_\mathrm{R} = 0$; iv) $\rho_\mathrm{T} = 0$, $\rho_\mathrm{R} = 0.7$; v) $\rho_\mathrm{T} = 0.5$, $\rho_\mathrm{R} = 0.7$; and vi) $\rho_\mathrm{T} = 0.9$, $\rho_\mathrm{R} = 0.7$. We can see that as κ grows, the asymptotic mean is approaching a deterministic quantity, independent of the transmit correlation and depends only on the receive correlation. This quantity is determined by the following corollary.

Corollary 1 (Maximum Scaling): If there exist the asymptotic EEDs F^{ψ_T} and F^{ψ_R}, then the asymptotic statistic

[9] The systems of equations in (19), (20) and (25), (26) can be solved numerically using a single standard function in a common mathematical software package such as MATHEMATICA, MAPLE or MATLAB. For example, we have used the library function '`Solve`' in MATHEMATICA.

$$\mu_{\text{asy}}\left(\bar{\gamma},\kappa\right) = 2\kappa\ln\left\{\frac{\sqrt{1+\zeta_2^{[t]}-2\rho_\mathrm{T}\zeta_2^{[t]}}+\sqrt{1+\zeta_2^{[t]}+2\rho_\mathrm{T}\zeta_2^{[t]}}}{2}\right\} + 2\ln\left\{\frac{\sqrt{1+\bar{\gamma}\zeta_1^{[t]}-2\rho_\mathrm{R}\bar{\gamma}\zeta_1^{[t]}}+\sqrt{1+\bar{\gamma}\zeta_1^{[t]}+2\rho_\mathrm{R}\bar{\gamma}\zeta_1^{[t]}}}{2}\right\} - \kappa\zeta_1^{[t]}\zeta_2^{[t]} \tag{23}$$

$$\sigma^2_{\text{asy}}\left(\bar{\gamma},\kappa\right) = -\ln\left\{1-\frac{1}{\kappa(\zeta_1^{[t]}\zeta_2^{[t]})^2}\times\left(1-\frac{1+3\zeta_2^{[t]}+2\left(1-2\rho_\mathrm{T}\right)\left(1+2\rho_\mathrm{T}\right)(\zeta_2^{[t]})^2}{\left[\left(1+\zeta_2^{[t]}-2\rho_\mathrm{T}\zeta_2^{[t]}\right)\left(1+\zeta_2^{[t]}+2\rho_\mathrm{T}\zeta_2^{[t]}\right)\right]^{3/2}}\right) \times\left(1-\frac{1+3\bar{\gamma}\zeta_1^{[t]}+2\left(1-2\rho_\mathrm{R}\right)\left(1+2\rho_\mathrm{R}\right)\bar{\gamma}^2(\zeta_1^{[t]})^2}{\left[\left(1+\bar{\gamma}\zeta_1^{[t]}-2\rho_\mathrm{R}\bar{\gamma}\zeta_1^{[t]}\right)\left(1+\bar{\gamma}\zeta_1^{[t]}+2\rho_\mathrm{R}\bar{\gamma}\zeta_1^{[t]}\right)\right]^{3/2}}\right)\right\} \tag{24}$$

where the quantities $\zeta_k^{[t]} \geq 0$, $k = 1,2$, are determined by solving the system of equations

$$\zeta_1^{[t]} = \frac{1}{\zeta_2^{[t]}}\left(1-\frac{1}{\sqrt{\left(1+\zeta_2^{[t]}-2\rho_\mathrm{T}\zeta_2^{[t]}\right)\left(1+\zeta_2^{[t]}+2\rho_\mathrm{T}\zeta_2^{[t]}\right)}}\right) \tag{25}$$

$$\zeta_2^{[t]} = \frac{1}{\kappa\zeta_1^{[t]}}\left(1-\frac{1}{\sqrt{\left(1+\bar{\gamma}\zeta_1^{[t]}-2\rho_\mathrm{R}\bar{\gamma}\zeta_1^{[t]}\right)\left(1+\bar{\gamma}\zeta_1^{[t]}+2\rho_\mathrm{R}\bar{\gamma}\zeta_1^{[t]}\right)}}\right). \tag{26}$$

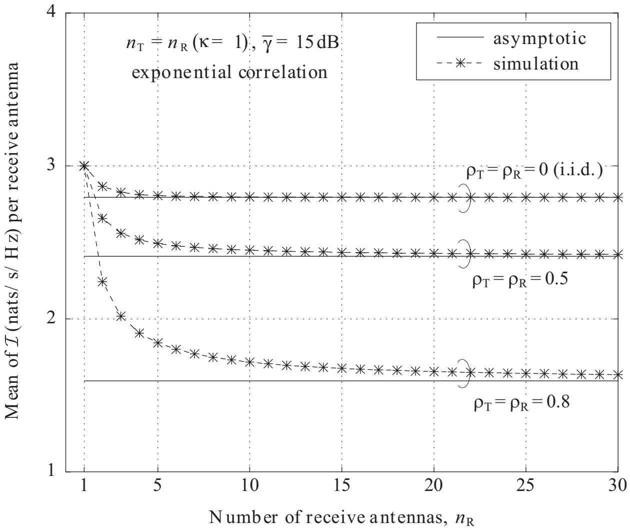

Fig. 1. Mean of $\mathcal{I}$ (nats/s/Hz) per receive antenna as a function of the number of receive antennas n_{R} and its asymptotic value for doubly correlated MIMO channels with exponential correlation $\mathbf{\Psi}_{\mathrm{T}} = \mathbf{\Psi}_{n_{\mathrm{T}}}\left(\psi_{\rho_{\mathrm{T}}}^{[\mathrm{e}]}\right)$ and $\mathbf{\Psi}_{\mathrm{R}} = \mathbf{\Psi}_{n_{\mathrm{R}}}\left(\psi_{\rho_{\mathrm{R}}}^{[\mathrm{e}]}\right)$. $n_{\mathrm{T}} = n_{\mathrm{R}}$ ($\kappa = 1$), $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$ (i.i.d.), 0.5, 0.8, and $\bar{\gamma} = 15$ dB.

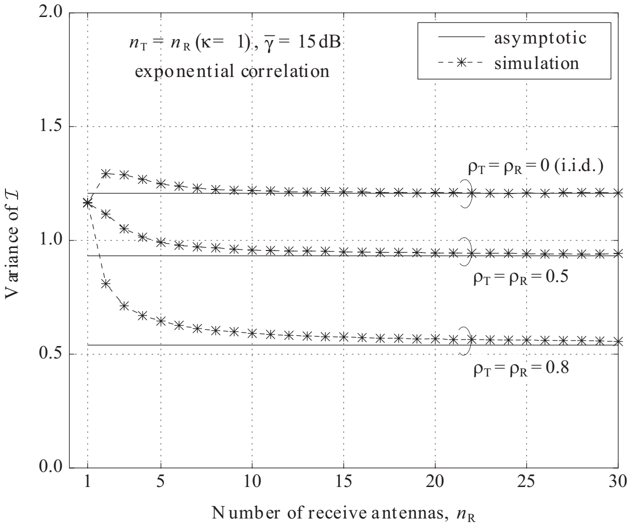

Fig. 2. Variance of $\mathcal{I}$ as a function of the number of receive antennas n_{R} and its asymptotic value for doubly correlated MIMO channels with exponential correlation $\mathbf{\Psi}_{\mathrm{T}} = \mathbf{\Psi}_{n_{\mathrm{T}}}\left(\psi_{\rho_{\mathrm{T}}}^{[\mathrm{e}]}\right)$ and $\mathbf{\Psi}_{\mathrm{R}} = \mathbf{\Psi}_{n_{\mathrm{R}}}\left(\psi_{\rho_{\mathrm{R}}}^{[\mathrm{e}]}\right)$. $n_{\mathrm{T}} = n_{\mathrm{R}}$ ($\kappa = 1$), $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$ (i.i.d.), 0.5, 0.8, and $\bar{\gamma} = 15$ dB.

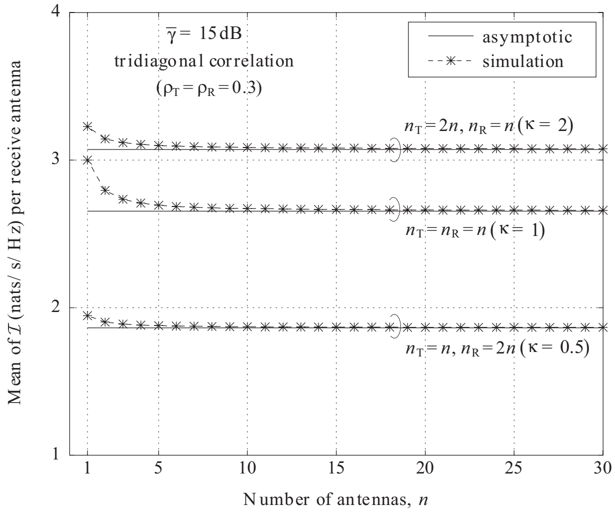

Fig. 3. Mean of $\mathcal{I}$ (nats/s/Hz) per receive antenna as a function of the number of antennas n and its asymptotic value for doubly correlated MIMO channels with tridiagonal correlation $\mathbf{\Psi}_{\mathrm{T}} = \mathbf{\Psi}_{n_{\mathrm{T}}}\left(\psi_{\rho_{\mathrm{T}}}^{[\mathrm{t}]}\right)$ and $\mathbf{\Psi}_{\mathrm{R}} = \mathbf{\Psi}_{n_{\mathrm{R}}}\left(\psi_{\rho_{\mathrm{R}}}^{[\mathrm{t}]}\right)$. $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0.3$ and $\bar{\gamma} = 15$ dB. $n_{\mathrm{T}} = n, n_{\mathrm{R}} = 2n$ ($\kappa = 0.5$); $n_{\mathrm{T}} = n_{\mathrm{R}} = n$ ($\kappa = 1$); and $n_{\mathrm{T}} = 2n, n_{\mathrm{R}} = n$ ($\kappa = 2$).

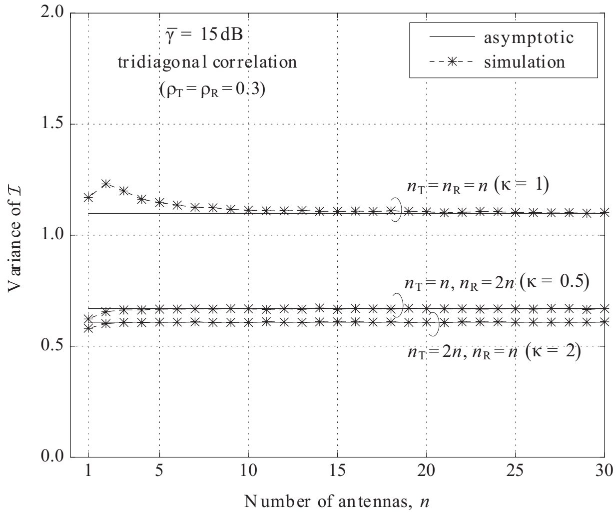

Fig. 4. Variance of $\mathcal{I}$ as a function of the number of antennas n and its asymptotic value for doubly correlated MIMO channels with tridiagonal correlation $\mathbf{\Psi}_{\mathrm{T}} = \mathbf{\Psi}_{n_{\mathrm{T}}}\left(\psi_{\rho_{\mathrm{T}}}^{[\mathrm{t}]}\right)$ and $\mathbf{\Psi}_{\mathrm{R}} = \mathbf{\Psi}_{n_{\mathrm{R}}}\left(\psi_{\rho_{\mathrm{R}}}^{[\mathrm{t}]}\right)$. $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0.3$ and $\bar{\gamma} = 15$ dB. $n_{\mathrm{T}} = n, n_{\mathrm{R}} = 2n$ ($\kappa = 0.5$); $n_{\mathrm{T}} = n_{\mathrm{R}} = n$ ($\kappa = 1$); and $n_{\mathrm{T}} = 2n, n_{\mathrm{R}} = n$ ($\kappa = 2$).

$\mu_{\text{asy}}\left(\bar{\gamma}, \kappa\right)$ for each SNR $\bar{\gamma}$ is a *monotonically increasing* function in $\kappa \geq 0$ and the *least* upper bound (or *supremum*) on the asymptotic growth rate of the mean with the number of receive antennas is given by

$$\sup_{\kappa \geq 0} \mu_{\text{asy}}\left(\bar{\gamma}, \kappa\right) = \Upsilon_0\left(\bar{\gamma}; \psi_{\mathrm{R}}\right). \tag{32}$$

Proof: See Appendix E. □

Note that κ is the limiting ratio related to the growth rates at which n_{T} and n_{R} tend to infinity. Hence, Corollary 1 implies that μ_{asy} increases with κ (i.e., increasing the growth rate of n_{T} and/or decreasing the growth rate of n_{R}), until it reaches its limit $\Upsilon_0\left(\bar{\gamma}; \psi_{\mathrm{R}}\right)$. The irrelevance of transmit correlation to the maximum scaling is due to the fact that as $\kappa \to \infty$, $\frac{1}{n_{\mathrm{T}}}\mathbf{H}_0\mathbf{D}_{\mathrm{T}}\mathbf{H}_0^{\dagger}$ converges almost surely to $\mathbf{I}_{n_{\mathrm{R}}}$ by the law of large numbers, where $\mathbf{D}_{\mathrm{T}}$ denotes the diagonal matrix of the eigenvalues of $\mathbf{\Psi}_{\mathrm{T}}$:

$$\begin{aligned}\lim_{\kappa \to \infty} \mu_{\text{asy}}\left(\bar{\gamma}, \kappa\right) &= \lim_{n_{\mathrm{R}} \to \infty} \frac{1}{n_{\mathrm{R}}} \ln\det\left(\mathbf{I}_{n_{\mathrm{R}}} + \bar{\gamma}\mathbf{\Psi}_{\mathrm{R}}\right)\\ &= \Upsilon_0\left(\bar{\gamma}; \psi_{\mathrm{R}}\right).\end{aligned}$$

From Corollary 1 and (55) in Appendix B, the asymptotic maximum growth rates for exponential receive correlation of $\rho_{\mathrm{R}} = 0$ and $\rho_{\mathrm{R}} = 0.7$ are equal to 3.485 and 2.867, respectively, as shown in Fig. 7.

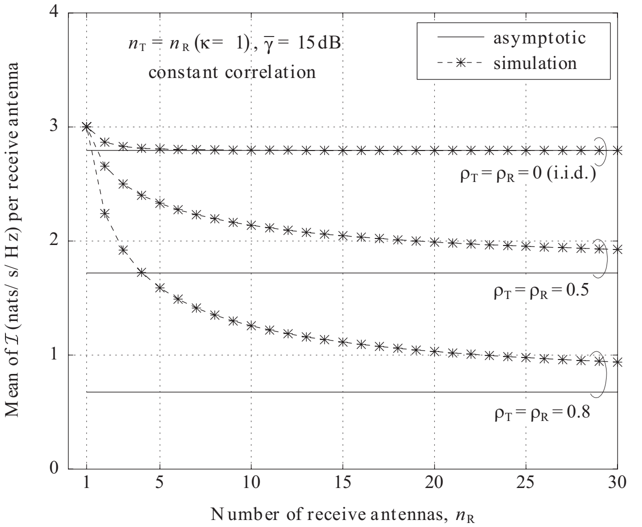

Fig. 5. Mean of $\mathcal{I}$ (nats/s/Hz) per receive antenna as a function of the number of receive antennas n_{R} and its asymptotic value for doubly correlated MIMO channels with constant correlation $\boldsymbol{\Psi}_{\mathrm{T}} = \boldsymbol{\Psi}_{n_{\mathrm{T}}}\left(\psi^{[c]}_{\rho_{\mathrm{T}}}\right)$ and $\boldsymbol{\Psi}_{\mathrm{R}} = \boldsymbol{\Psi}_{n_{\mathrm{R}}}\left(\psi^{[c]}_{\rho_{\mathrm{R}}}\right)$. $n_{\mathrm{T}} = n_{\mathrm{R}}$ ($\kappa = 1$), $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$ (i.i.d.), 0.5, 0.8, and $\bar{\gamma} = 15$ dB.

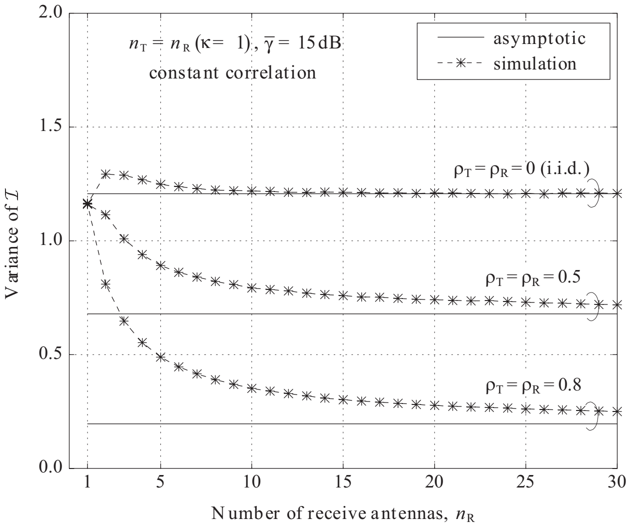

Fig. 6. Variance of $\mathcal{I}$ as a function of the number of receive antennas n_{R} and its asymptotic value for doubly correlated MIMO channels with constant correlation $\boldsymbol{\Psi}_{\mathrm{T}} = \boldsymbol{\Psi}_{n_{\mathrm{T}}}\left(\psi^{[c]}_{\rho_{\mathrm{T}}}\right)$ and $\boldsymbol{\Psi}_{\mathrm{R}} = \boldsymbol{\Psi}_{n_{\mathrm{R}}}\left(\psi^{[c]}_{\rho_{\mathrm{R}}}\right)$. $n_{\mathrm{T}} = n_{\mathrm{R}}$ ($\kappa = 1$), $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$ (i.i.d.), 0.5, 0.8, and $\bar{\gamma} = 15$ dB.

C. Asymptotic Peak Variance

Fig. 8 shows the asymptotic variance $\sigma^2_{\mathrm{asy}}(\bar{\gamma}, \kappa)$ as a function of the ratio κ for $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$, 0.5, and 0.7 when $\bar{\gamma} = 20$ dB and $\bar{\gamma} = -10$ dB. As can be seen from Fig. 8, the asymptotic statistic $\sigma^2_{\mathrm{asy}}(\bar{\gamma}, \kappa)$ is a *unimodal* function in $\kappa \geq 0$ and hence, there exists a unique value—the mode κ_{mode}—of the ratio κ that maximizes the asymptotic variance $\sigma^2_{\mathrm{asy}}(\bar{\gamma}, \kappa)$ for each SNR $\bar{\gamma}$. For example, the maximum variation of the asymptotic mutual information, i.e., asymptotic peak variance for i.i.d. MIMO channels ($\psi_{\mathrm{T}} = \psi_{\mathrm{R}} = \psi_{\delta}$) is given by

$$\max_{\kappa \geq 0} \sigma^2_{\mathrm{asy}}(\bar{\gamma}, \kappa) = \ln\left(\frac{\sqrt{1+\bar{\gamma}}+1}{2}\right) \tag{33}$$

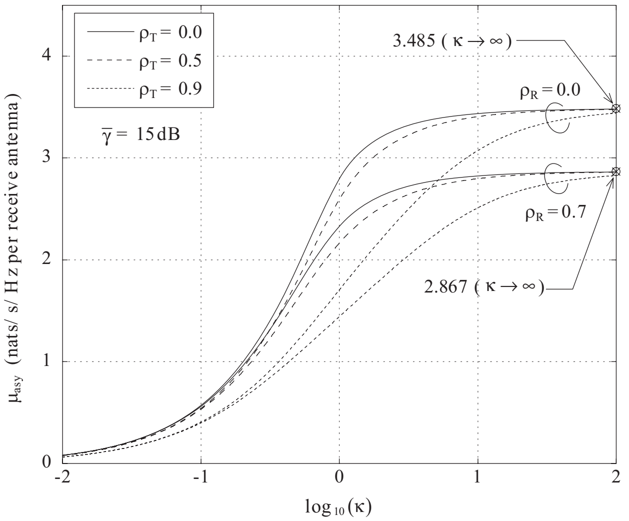

Fig. 7. Asymptotic mean $\mu_{\mathrm{asy}}(\bar{\gamma}, \kappa)$ (nats/s/Hz per receive antenna) as a function of the ratio κ for doubly correlated MIMO channels with exponential correlation. $\bar{\gamma} = 15$ dB.

with the mode

$$\kappa_{\mathrm{mode}} = \frac{\bar{\gamma}}{1+\bar{\gamma}} \tag{34}$$

which is the solution of $\partial\sigma^2_{\mathrm{asy}}(\bar{\gamma}, \kappa)/\partial\kappa = 0$. The mode (34) reveals that the asymptotic variance of $\mathcal{I}$ reaches its maximum around $\kappa = 1$ (symmetric antenna configuration) at high SNR, but around $\kappa = \bar{\gamma}$ ($\ll 1$) at low SNR. In Fig. 8, we can determine the mode κ_{mode} numerically. When $\bar{\gamma} = 20$ dB (high SNR), $\kappa_{\mathrm{mode}} = 0.990$, 0.973, and 0.915 for $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$, 0.5, and 0.7, respectively. In contrast, when $\bar{\gamma} = -10$ dB (low SNR), $\kappa_{\mathrm{mode}} = 0.091$, 0.115, and 0.175 for $\rho_{\mathrm{T}} = \rho_{\mathrm{R}} = 0$, 0.5, and 0.7, respectively.

It can be also observed that the effect of fading correlation on the variance of $\mathcal{I}$ is different in high- and low-SNR regimes. When $\bar{\gamma} = 20$ dB (high SNR), larger fading correlation leads to an increase in the variance under highly asymmetric antenna configuration ($\kappa \gg 1$ or $\kappa \ll 1$), but a decrease in the variance for a symmetric case ($n_{\mathrm{T}} \approx n_{\mathrm{R}}$). In contrast, when $\bar{\gamma} = -10$ dB (low SNR), larger fading correlation increases the variance for any value of κ.[10]

V. Conclusions

We derived the statistics of mutual information for doubly correlated MIMO channels in the asymptotic regime where the number of transmit and receive antennas goes to infinity. The asymptotic mean (per receive antenna) and variance of the mutual information with equal-power allocation have been obtained in terms of the asymptotic linear spectral statistics—the correlants—for structured spatial correlation matrices. As an example, we presented asymptotic mutual information for special classes of Toeplitz correlation matrices

[10]This observation is related to the following Schur monotonicity property of the low-SNR variance: the variance of $\mathcal{I}$ as functionals of the eigenvalues of correlation matrices is *monotonically increasing in a sense of Schur* at low SNR (the proof can be found in [24, Appendix C]). See [25] for more details about Schur monotonicity related to correlation matrices.

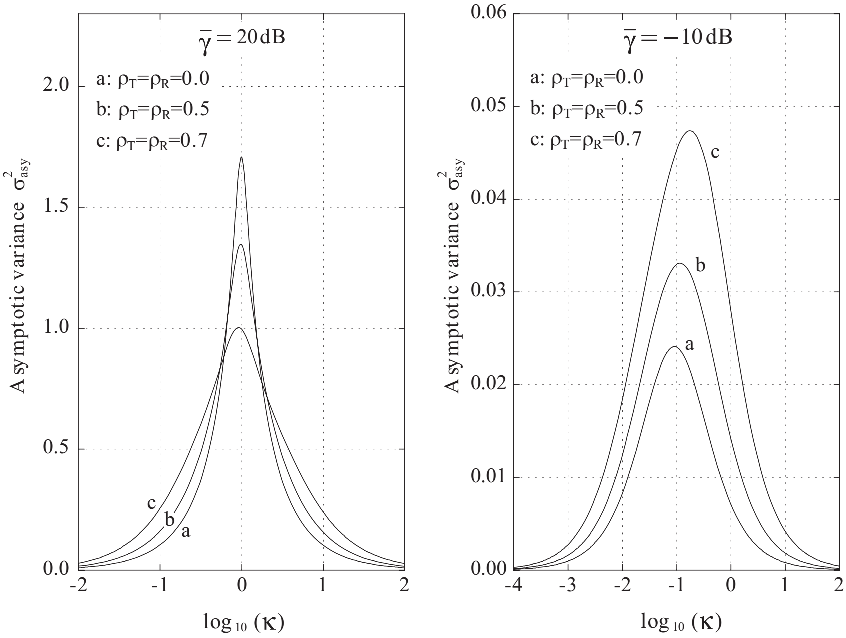

Fig. 8. Asymptotic variance $\sigma^2_{\text{asy}}(\bar{\gamma}, \kappa)$ as a function of the ratio κ for doubly correlated MIMO channels with exponential correlation when $\bar{\gamma} = 20$ dB and $\bar{\gamma} = -10$ dB. $\rho_{\text{T}} = \rho_{\text{R}} = 0$ (i.i.d.), 0.5, and 0.7.

(exponential, tridiagonal, and constant correlation matrices). From monotone or unimodal properties of the asymptotic statistics with respect to the ratio $n_{\text{T}}/n_{\text{R}}$, we determined the maximum scaling (with the number of receive antennas) and the maximum variation of the mutual information in the asymptotic regime with the following observations:

- The maximum scaling of the mutual information with the number of receive antennas is independent of transmit correlation and depends only on receive correlation.
- The peak variance of the mutual information occurs at $n_{\text{T}} \approx n_{\text{R}}$ in a high-SNR regime, but at $n_{\text{T}} \approx \bar{\gamma} \cdot n_{\text{R}}$ in a low-SNR regime.

Our asymptotic results provide insights complementary to the previous exact analysis for the finite number of antennas [6].

APPENDIX

A. Proofs of Lemmas 1 and 2

The fundamental eigenvalue distribution theorem of Szegö (see [19, Ch. 5]) states that if $\{c_k\}_{k=-\infty}^{\infty}$ are Fourier coefficients of a bounded real-valued function ϕ (Riemann integrable on $[-\pi, \pi]$), then the eigenvalues of $n \times n$ Hermitian Toeplitz matrices $\mathbf{T}_n(\phi) = [c_{i-j}]_{i,j=1,2,\ldots,n}$ and the values of ϕ at n equally-spaced points in the interval $[-\pi, \pi]$ are equally distributed in a sense of Weyl [19, Sec. 5.1], as $n \to \infty$.[11] For conceptual simplicity, the condition for Riemann integrability of ϕ has been replaced with the absolute summability of $\{c_k\}_{k=-\infty}^{\infty}$ in [20] (see also [21]) at the expense of mathematical sophistication and generality. Lemmas 1 and 2 follow directly from this Szegö's theorem.

1) Proof of Lemma 1: Let $\phi^{[e]}$ be the Fourier series of the sequence $\left\{\rho^{|k|}\right\}_{k=-\infty}^{\infty}$, i.e.,

$$\phi^{[e]}(\theta) = \sum_{k=-\infty}^{\infty} \rho^{|k|} e^{jk\theta} = \frac{1-\rho^2}{1-2\rho\cos\theta+\rho^2} \tag{35}$$

where $j = \sqrt{-1}$. Since the sequence $\left\{\rho^{|k|}\right\}_{k=-\infty}^{\infty}$ for $\rho \in [0, 1)$ is absolutely summable, the asymptotic EED $F^{\psi_\rho^{[e]}}$ exists (due to Szegö's theorem) and is given by [21, Corollary 4.1]

$$F^{\psi_\rho^{[e]}}(\lambda) = \frac{1}{2\pi} \int_{\Theta_\lambda = \{\theta \in [-\pi,\pi]:\, \phi^{[e]}(\theta) \le \lambda\}} d\theta. \tag{36}$$

Using (35), $F^{\psi_\rho^{[e]}}$ can be evaluated as (37), shown at the top of the next page, where

$$\lambda_{\min}\left(\psi_\rho^{[e]}\right) = \min_{-\pi \le \theta \le \pi} \phi^{[e]}(\theta) = \frac{1-\rho}{1+\rho} \tag{38}$$

$$\lambda_{\max}\left(\psi_\rho^{[e]}\right) = \max_{-\pi \le \theta \le \pi} \phi^{[e]}(\theta) = \frac{1+\rho}{1-\rho}. \tag{39}$$

Now, taking the asymptotic eigenvalue density of $\mathbf{\Psi}_n\left(\psi_\rho^{[e]}\right)$ to be the derivative of $F^{\psi_\rho^{[e]}}(\lambda)$ with respect to λ wherever it exists—which is everywhere except at $\lambda_{\min}\left(\psi_\rho^{[e]}\right)$ and $\lambda_{\max}\left(\psi_\rho^{[e]}\right)$—and defining the density to be zero at $\lambda_{\min}\left(\psi_\rho^{[e]}\right)$ and $\lambda_{\max}\left(\psi_\rho^{[e]}\right)$ give the result (16).

[11] The sequences $\{a_\ell\}_{\ell=1}^{n}$ and $\{b_\ell\}_{\ell=1}^{n}$ with $a_\ell, b_\ell \in [m, M]$ are said to be equally distributed in Weyl's sense in the interval $[m, M]$ as $n \to \infty$, if

$$\lim_{n\to\infty} \frac{1}{n} \sum_{\ell=1}^{n} [G(a_\ell) - G(b_\ell)] = 0$$

for any function G continuous on the interval $[m, M]$.

$$F^{\psi_\rho^{[e]}}(\lambda) = \begin{cases} 0, & \lambda < \lambda_{\min}\left(\psi_\rho^{[e]}\right) \\ 1 - \frac{1}{\pi}\cos^{-1}\left(\frac{1+\rho^2}{2\rho} - \frac{1-\rho^2}{2\rho\lambda}\right), & \lambda_{\min}\left(\psi_\rho^{[e]}\right) \le \lambda \le \lambda_{\max}\left(\psi_\rho^{[e]}\right) \\ 1, & \lambda > \lambda_{\max}\left(\psi_\rho^{[e]}\right) \end{cases} \tag{37}$$

$$F^{\psi_\rho^{[t]}}(\lambda) = \begin{cases} 0, & \lambda < \lambda_{\min}\left(\psi_\rho^{[t]}\right) \\ 1 - \frac{1}{\pi}\cos^{-1}\left(\frac{\lambda-1}{2\rho}\right), & \lambda_{\min}\left(\psi_\rho^{[t]}\right) \le \lambda \le \lambda_{\max}\left(\psi_\rho^{[t]}\right) \\ 1, & \lambda > \lambda_{\max}\left(\psi_\rho^{[t]}\right) \end{cases} \tag{42}$$

2) Proof of Lemma 2: Similar to the proof of Lemma 1, let $\phi^{[t]}$ be the Fourier series of the sequence $\left\{c_k^{[t]}\right\}_{k=-\infty}^{\infty}$ with

$$c_k^{[t]} = \begin{cases} 1, & \text{if } k = 0 \\ \rho & \text{if } k = \pm 1 \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

Then,

$$\phi^{[t]}(\theta) = 1 + 2\rho\cos\theta. \tag{41}$$

Since the sequence $\left\{c_k^{[t]}\right\}_{k=-\infty}^{\infty}$ for $\rho \in [0, 0.5)$ is absolutely summable, the asymptotic EED $F^{\psi_\rho^{[t]}}$ can be found as (42), shown at the top of the page, where

$$\lambda_{\min}\left(\psi_\rho^{[t]}\right) = \min_{-\pi\le\theta\le\pi} \phi^{[t]}(\theta) = 1 - 2\rho \tag{43}$$

$$\lambda_{\max}\left(\psi_\rho^{[t]}\right) = \max_{-\pi\le\theta\le\pi} \phi^{[t]}(\theta) = 1 + 2\rho. \tag{44}$$

Now, taking the asymptotic eigenvalue density of $\mathbf{\Psi}_n\left(\psi_\rho^{[t]}\right)$ to be the derivative of $F^{\psi_\rho^{[t]}}(\lambda)$ with respect to λ wherever it exists and defining the density to be zero at $\lambda_{\min}\left(\psi_\rho^{[t]}\right)$ and $\lambda_{\max}\left(\psi_\rho^{[t]}\right)$ give the result (22).

B. Proof of Theorem 1

Using (6) and Lemma 1, the zeroth-order correlant $\Upsilon_0\left(\xi; \psi_\rho^{[e]}\right)$ can be written as

$$\Upsilon_0\left(\xi; \psi_\rho^{[e]}\right) = \int_{\frac{1-\rho}{1+\rho}}^{\frac{1+\rho}{1-\rho}} \frac{\ln(1+\xi\lambda)\, d\lambda}{\pi\lambda\sqrt{\left(\lambda - \frac{1-\rho}{1+\rho}\right)\left(\frac{1+\rho}{1-\rho} - \lambda\right)}}. \tag{45}$$

Making the change of variable

$$\lambda = \frac{1+\rho^2 - 2\rho\cos\theta}{1-\rho^2},$$

we obtain

$$\Upsilon_0\left(\xi; \psi_\rho^{[e]}\right) = \frac{1}{2\pi}\int_0^{2\pi} \frac{1-\rho^2}{1+\rho^2-2\rho\cos\theta} \times \ln\left\{1 + \frac{\xi\left(1+\rho^2-2\rho\cos\theta\right)}{1-\rho^2}\right\} d\theta. \tag{46}$$

The integral (46) is related to Poisson's integral formula for a disk [26, Theorem 22]:

$$u(z) = \frac{1}{2\pi}\int_0^{2\pi} P(r_0, r, \theta - \alpha)\, u\big(r_0 e^{j\theta}\big)\, d\theta \tag{47}$$

where $u(z)$ is a harmonic function on the open disk in $\mathbb{C}$, $z = re^{j\alpha}$ with $r \in [0, r_0)$, and $P(r_0, r, \theta-\alpha)$ is the Poisson kernel defined by

$$P(r_0, r, \theta - \alpha) \triangleq \frac{r_0^2 - r^2}{r_0^2 - 2r_0 r\cos(\theta-\alpha) + r^2}. \tag{48}$$

Let $\varphi(z) \triangleq 2\ln(r_1 - r_2 z)$ where $r_1, r_2 \ge 0$. Then, it is easy to show that the function φ is holomorphic (or analytic) on the set $\mathbb{C} - \left\{z \in \mathbb{R} : z \ge \frac{r_1}{r_2}\right\}$ and that

$$\mathfrak{Re}\left\{\varphi\left(e^{j\theta}\right)\right\} = \ln\left|r_1 - r_2 e^{j\theta}\right|^2. \tag{49}$$

Since the real part of a holomorphic function is harmonic, it follows from (47) with $r_0 = 1$ that

$$\Upsilon_0\left(\xi; \psi_\rho^{[e]}\right) = \varphi(\rho) = 2\ln(r_1 - r_2\rho) \tag{50}$$

where

$$r_1^2 + r_2^2 = 1 + \frac{\xi\left(1+\rho^2\right)}{1-\rho^2} \tag{51}$$

$$r_1 r_2 = \frac{2\xi\rho}{1-\rho^2} \tag{52}$$

with $\frac{r_1}{r_2} \ge 1$. By solving for r_1 and r_2, we get

$$r_1 = \frac{1}{2}\left(\sqrt{\tfrac{1+\rho}{1-\rho}\left(\xi + \tfrac{1-\rho}{1+\rho}\right)} + \sqrt{\tfrac{1-\rho}{1+\rho}\left(\xi + \tfrac{1+\rho}{1-\rho}\right)}\right) \tag{53}$$

$$r_2 = \frac{1}{2}\left(\sqrt{\tfrac{1+\rho}{1-\rho}\left(\xi + \tfrac{1-\rho}{1+\rho}\right)} - \sqrt{\tfrac{1-\rho}{1+\rho}\left(\xi + \tfrac{1+\rho}{1-\rho}\right)}\right). \tag{54}$$

Substituting (53) and (54) into (50) yields

$$\Upsilon_0\left(\xi; \psi_\rho^{[e]}\right) = 2\ln\left(\frac{\sqrt{\frac{1-\rho}{1+\rho} + \xi} + \sqrt{\frac{1+\rho}{1-\rho} + \xi}}{\sqrt{\frac{1-\rho}{1+\rho}} + \sqrt{\frac{1+\rho}{1-\rho}}}\right). \tag{55}$$

By definition, for $\ell \in \mathbb{N}$, we have

$$\Upsilon_\ell\left(\xi;\psi_\rho^{[e]}\right) = \frac{2(-1)^{\ell-1}}{(\ell-1)!}\frac{d^\ell}{d\xi^\ell}\ln\left(\sqrt{\tfrac{1-\rho}{1+\rho}+\xi}+\sqrt{\tfrac{1+\rho}{1-\rho}+\xi}\right). \tag{56}$$

In particular, the first and second order correlants are given respectively by

$$\Upsilon_1\left(\xi;\psi_\rho^{[e]}\right) = \frac{1}{\sqrt{\left(\frac{1-\rho}{1+\rho}+\xi\right)\left(\frac{1+\rho}{1-\rho}+\xi\right)}} \tag{57}$$

$$\Upsilon_2\left(\xi;\psi_\rho^{[e]}\right) = \frac{\frac{1-\rho}{1+\rho}+\frac{1+\rho}{1-\rho}+2\xi}{2\left[\left(\frac{1-\rho}{1+\rho}+\xi\right)\left(\frac{1+\rho}{1-\rho}+\xi\right)\right]^{3/2}}. \tag{58}$$

Finally, combining Proposition 1, (55), (57), and (58) completes the proof.

C. Proof of Theorem 2

Using (6) and Lemma 2, the zeroth-order correlant $\Upsilon_0\left(\xi;\psi_\rho^{[t]}\right)$ can be written as

$$\Upsilon_0\left(\xi;\psi_\rho^{[t]}\right) = \int_{1-2\rho}^{1+2\rho}\frac{\ln(1+\xi\lambda)\,d\lambda}{\pi\sqrt{(\lambda-1+2\rho)(1+2\rho-\lambda)}}. \tag{59}$$

Making the change of variable $\lambda = 1-2\rho\cos\theta$, we obtain

$$\Upsilon_0\left(\xi;\psi_\rho^{[t]}\right) = \frac{1}{2\pi}\int_0^{2\pi}\ln(1+\xi-2\xi\rho\cos\theta)\,d\theta. \tag{60}$$

To evaluate the integral (60), we use the same steps leading to (55). Noting that the Poisson kernel in this case is $P(r_0,r,\theta-\alpha)=1$, we have $r_0=1$, $r=0$, and

$$\Upsilon_0\left(\xi;\psi_\rho^{[t]}\right) = \varphi(0) = 2\ln(r_1) \tag{61}$$

where

$$r_1^2+\frac{\xi^2\rho^2}{r_1^2} = 1+\xi \tag{62}$$

with $r_1 \geq 0$. It is easy to show that

$$r_1 = \frac{1}{2}\left(\sqrt{1+\xi-2\xi\rho}+\sqrt{1+\xi+2\xi\rho}\right). \tag{63}$$

Hence, we get

$$\Upsilon_0\left(\xi;\psi_\rho^{[t]}\right) = 2\ln\left(\frac{\sqrt{1+\xi-2\xi\rho}+\sqrt{1+\xi+2\xi\rho}}{2}\right) \tag{64}$$

$$\Upsilon_1\left(\xi;\psi_\rho^{[t]}\right) = \frac{1}{\xi}\left(1-\frac{1}{\sqrt{(1+\xi-2\xi\rho)(1+\xi+2\xi\rho)}}\right) \tag{65}$$

$$\Upsilon_2\left(\xi;\psi_\rho^{[t]}\right) = \frac{1}{\xi^2}\left(1-\frac{1+3\xi+2(1-2\rho)(1+2\rho)\xi^2}{[(1+\xi-2\xi\rho)(1+\xi+2\xi\rho)]^{3/2}}\right) \tag{66}$$

from which and Proposition 1, we complete the proof.

D. Proof of Theorem 3

It is clear from Definition 2 that for $\xi > 0$,

$$\Upsilon_\ell\left(\xi;\psi_\rho^{[c]}\right) = \mathcal{K}_\ell(1-\rho,\xi). \tag{67}$$

Hence, from (13), (14), and (67), we arrive at the following system of equations for $\zeta_k^{[c]} \geq 0$, $k=1,2$:

$$\zeta_1^{[c]} = \frac{1-\rho_{\mathrm{T}}}{1+\zeta_2^{[c]}(1-\rho_{\mathrm{T}})} \tag{68}$$

$$\zeta_2^{[c]} = \frac{\bar{\gamma}}{\varrho}\cdot\frac{1-\rho_{\mathrm{R}}}{1+\bar{\gamma}\zeta_1^{[c]}(1-\rho_{\mathrm{R}})}. \tag{69}$$

Solving for $\zeta_1^{[c]}$ and $\zeta_2^{[c]}$, and after some algebra, we obtain

$$\zeta_1^{[c]} = (1-\rho_{\mathrm{T}})\left[1-\mathcal{B}(\bar{\gamma},\varrho_{\mathrm{c}},\kappa)\right] \tag{70}$$

$$\zeta_2^{[c]} = (1-\rho_{\mathrm{R}})\left[\frac{\bar{\gamma}}{\kappa}-\bar{\gamma}\mathcal{B}(\bar{\gamma},\varrho_{\mathrm{c}},\kappa)\right] \tag{71}$$

$$\zeta_1^{[c]}\zeta_2^{[c]} = \mathcal{B}(\bar{\gamma},\varrho_{\mathrm{c}},\kappa). \tag{72}$$

Combining Proposition 1, (67), and (70)–(72) completes the proof.

E. Proof of Corollary 1

It is easy to show from (6) and Proposition 1 that

$$\begin{aligned}\frac{\partial\mu_{\mathrm{asy}}(\bar{\gamma},\kappa)}{\partial\kappa} &= \Upsilon_0(\zeta_2;\psi_{\mathrm{T}})-\zeta_1\zeta_2\\ &= \Upsilon_0(\zeta_2;\psi_{\mathrm{T}})-\Upsilon_1(\zeta_2;\psi_{\mathrm{T}})\zeta_2\\ &= \int_{\lambda_{\min}(\psi_{\mathrm{T}})}^{\lambda_{\max}(\psi_{\mathrm{T}})}\left\{\ln(1+\zeta_2\lambda)-\frac{\zeta_2\lambda}{1+\zeta_2\lambda}\right\}dF^{\psi_{\mathrm{T}}}(\lambda)\end{aligned} \tag{73}$$

Since $\ln(1+x) \geq \frac{x}{1+x}$ for $x \geq 0$, it follows from (73) that

$$\frac{\partial\mu_{\mathrm{asy}}(\bar{\gamma},\kappa)}{\partial\kappa} \geq 0 \tag{74}$$

which means that $\mu_{\mathrm{asy}}(\bar{\gamma},\kappa)$ is a monotonically increasing function in $\kappa \geq 0$. Hence,

$$\sup_{\kappa\geq 0}\mu_{\mathrm{asy}}(\bar{\gamma},\kappa) = \lim_{\kappa\to\infty}\mu_{\mathrm{asy}}(\bar{\gamma},\kappa). \tag{75}$$

It is clear that if $\kappa\to\infty$, then $\zeta_1\to 1$, $\zeta_2\to 0$, and

$$\begin{aligned}\lim_{\kappa\to\infty}\mu_{\mathrm{asy}}(\bar{\gamma},\kappa) &= \lim_{\kappa\to\infty}\frac{\Upsilon_0(\zeta_2;\psi_{\mathrm{T}})-\zeta_1\zeta_2}{1/\kappa}+\Upsilon_0(\bar{\gamma};\psi_{\mathrm{R}})\\ &= \underbrace{\left[-\kappa^2\left(-\frac{\partial\zeta_1}{\partial\kappa}\cdot\zeta_2\right)\right]\Bigg|_{\kappa=\infty}}_{=0}+\Upsilon_0(\bar{\gamma};\psi_{\mathrm{R}})\end{aligned} \tag{76}$$

where

$$\frac{\partial\zeta_1}{\partial\kappa} = \frac{\zeta_2\Upsilon_2(\zeta_2;\psi_{\mathrm{T}})}{\kappa}\left[1-\frac{\bar{\gamma}^2}{\kappa}\Upsilon_2(\zeta_2;\psi_{\mathrm{T}})\Upsilon_2(\bar{\gamma}\zeta_1;\psi_{\mathrm{R}})\right]^{-1}. \tag{77}$$

From (75) and (76), we arrive at the desired result (32).

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