Schur Complement Based Analysis of MIMO Zero-Forcing for Rician Fading

Constantin Siriteanu, Akimichi Takemura, Satoshi Kuriki, Donald St. P. Richards, and Hyundong Shin

***Abstract*—For multiple-input/multiple-output (MIMO) spatial multiplexing with zero-forcing detection (ZF), signal-to-noise ratio (SNR) analysis for Rician fading involves the cumbersome noncentral-Wishart distribution (NCWD) of the transmit sample–correlation (Gramian) matrix. An *approximation* with a *virtual* CWD previously yielded for the ZF SNR an approximate (virtual) Gamma distribution. However, analytical conditions qualifying the accuracy of the SNR-distribution approximation were unknown. Therefore, we have been attempting to exactly characterize ZF SNR for Rician fading. Our previous attempts succeeded only for the sole Rician-fading stream under Rician–Rayleigh fading, by writing the ZF SNR as scalar Schur complement (SC) in the Gramian. Herein, we pursue a more general matrix-SC-based analysis to characterize SNRs when several streams may undergo Rician fading. On one hand, for full-Rician fading, the SC distribution is found to be exactly a CWD if and only if a channel-mean–correlation *condition* holds. Interestingly, this CWD then coincides with the *virtual* CWD ensuing from the *approximation*. Thus, under the *condition*, the actual and virtual SNR-distributions coincide. On the other hand, for Rician–Rayleigh fading, the matrix-SC distribution is characterized in terms of the determinant of a matrix with elementary-function entries, which also yields a new characterization of the ZF SNR. Average error probability results validate our analysis vs. simulation.**

***Index Terms*—MIMO, non/central-Wishart matrix distribution, Rayleigh and Rician (Ricean) fading, schur complement, zero-forcing.**

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C. Siriteanu is with the Graduate School of Information Science and Technology, Osaka University, Osaka, Japan (e-mail: constantin.siriteanu@ist.osaka-u.ac.jp).

A. Takemura is with the Department of Mathematical Informatics, University of Tokyo, Japan.

S. Kuriki is with the Institute of Statistical Mathematics, Tachikawa, Tokyo, Japan.

D. St. P. Richards is with the Department of Statistics, Pennsylvania State University, University Park, Pennsylvania, USA.

H. Shin is with the Department of Electronics and Radio Engineering, Kyung Hee University, Gyeonggi-do 446-701, South Korea (e-mail: hshin@khu.ac.kr).

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I. Introduction

A. Background, Motivation, Scope, and Main Assumptions

MULTIPLE-INPUT/MULTIPLE-OUTPUT (MIMO) communications principles have maintained substantial research interest [1]–[5] and have also been adopted in standards [6], [7]. However, gaps remain in our ability to evaluate MIMO performance, based on analysis, for realistic channel propagation conditions and relatively simple transceiver processing: e.g., for MIMO spatial-multiplexing for Rician fading and linear detection methods, such as zero-forcing detection (ZF) [8], [9] or minimum mean-square-error detection (MMSE) [10].

Rician fading is both theoretically more general and practically more realistic than Rayleigh fading (which yields simpler analysis), according to the state-of-the-art WINNER II channel model [11]. ZF has relatively-low implementation complexity, and, thus, is attractive for practical implementation, as recently acknowledged under the massive-MIMO framework [4], [5].

Herein, we study MIMO ZF[1] under Rician and Rayleigh fading mixtures that (beside promoting analysis tractability) may occur in macrocells, microcells, and heterogeneous networks—see [10], [12] and relevant references therein.

We consider a MIMO system whereby the symbol streams transmitted from N_{T} antennas are received with $N_{\mathrm{R}} \geq N_{\mathrm{T}}$ antennas. The $N_{\mathrm{R}} \times N_{\mathrm{T}}$ channel matrix $\mathbf{H}$ is assumed Gaussian-distributed. For analysis tractability, we assume that elements on different rows of $\mathbf{H}$ are uncorrelated, and that each of the N_{R} rows of $\mathbf{H}$ has the same covariance matrix. Then, given the transmit sample-correlation matrix $\mathbf{W} = \mathbf{H}^{\mathcal{H}}\mathbf{H}$, also known as Gramian matrix [13, p. 288], the ZF signal-to-noise ratio (SNR) for a stream is determined by the corresponding diagonal element of $\mathbf{W}^{-1}$ [14, Eq. (5)].

B. Previous Work on MIMO ZF for Rician Fading

For MIMO ZF under Rayleigh-only fading, the stream SNRs have been shown to be Gamma-distributed in [14], based on the fact that, when the mean $\mathbf{H}_{\mathrm{d}}$ of $\mathbf{H}$ is zero, $\mathbf{W}$ has a central-Wishart distribution[2] (CWD) [15], and then $\mathbf{W}^{-1}$ has a known inverse-CWD [16, p. 97].

On the other hand, under Rician fading, i.e., when $\mathbf{H}_{\mathrm{d}} \neq \mathbf{0}$, $\mathbf{W}$ is NCWD [15], [17], and then $\mathbf{W}^{-1}$ has an unknown distribution. Therefore, for MIMO ZF under full-Rician

[1] A study of MMSE is left for future work.

[2] For simplicity, N/CWD stand herein for both "non/central-Wishart distribution" and "non/central-Wishart-distributed."

fading,[3] we attempted in [9] to characterize the ZF SNR distribution by approximating the actual NCWD of the Gramian matrix $\mathbf{W}$ with a *virtual* CWD of equal mean. This approximation had been proposed in [18] and had been exploited for MIMO ZF analysis several times [9, Refs. 24–27, 30, 31], because it yields a simple virtual Gamma distribution to approximate the unknown actual distribution of the ZF SNR.

However, the accuracy of this SNR-distribution approximation has been qualified only empirically. Thus, numerical results shown — without explanation or support — mostly for $\mathbf{H}_{\rm d}$ of rank $r = 1$ obtained as outer-product of receive and transmit array-steering vectors in [9, Refs. 24–27, 30, 31] found the approximation reliable. In [9], we also found it most accurate for such rank-1 $\mathbf{H}_{\rm d}$; higher r yielded poorer accuracy, and $r = N_{\rm T}$ made the approximation unusable. Finally, we also revealed in [9] that, even for $r = 1$, the approximation accuracy depends on the mean-correlation combination.

Because [9] found the ZF SNR distribution approximation not consistently reliable and because analytical conditions for its accuracy were unknown, we pursued in [12] an exact ZF-SNR analysis. That analysis was found tractable only for the case when the intended Stream 1 undergoes Rician fading whereas interfering Streams $2, \cdots, N_{\rm T}$ undergo Rayleigh fading, i.e., Rician(1)/Rayleigh$(N_{\rm T} - 1)$ fading, for which, incidentally, $r = 1$. Proceeding in [12] from the *vector-matrix* partitioning according to fading types $\mathbf{H} = (\mathbf{h}_1 \quad \mathbf{H}_2)$, we could write the Stream-1 ZF SNR in terms of the scalar Schur complement (SC) [19], [20], [13, Sec. 3.4] of submatrix $\mathbf{W}_{22} = \mathbf{H}_2^{\mathcal{H}} \mathbf{H}_2$ in the NCWD Gramian matrix $\mathbf{W}$.

Note that the SC arises "naturally" in statistical analyses as the (sample) correlation matrix of the conditioned Gaussian distribution [19, p. 186], as also exemplified in [12] and this paper. Also, after minimizing a Hermitian form over some variables, the matrix in the ensuing Hermitian form is a SC [13, Eqs. (A.13–14), p. 650].

In [12, Eq. (9)], we recast the scalar SC as a Hermitian form whereby the vector and matrix correspond, respectively, to the intended and interfering streams, based on [8, Eq. (7)]. By first conditioning on and then averaging over $\mathbf{H}_2$, we expressed exactly, in [12, Eq. (31)], the moment generating function (m.g.f.) of the ZF SNR for the Rician-fading Stream 1, in terms of the confluent hypergeometric function ${}_1F_1(N; N_{\rm R}; \sigma_1)$, where $N = N_{\rm R} - N_{\rm T} + 1$, and scalar σ_1 is a function of channel mean and transmit-correlation.[4]

Finally, average error probability (AEP) results shown in [12, Figs. 1 and 2] for Rician(1)/Rayleigh $(N_{\rm T} - 1)$ fading further supported our observation from [9] that the actual-virtual SNR-distribution approximation is inconsistently accurate even for $r = 1$. On the other hand, results shown in [12, Fig. 10] for Rayleigh(1)/Rician $(N_{\rm T} - 1)$ fading suggested that the approximation can be very accurate even for $r > 1$. The empirical observations from [9], [12] prompted us to seek the analytical condition that renders this approximation exact.

[3] I.e., all streams undergo Rician fading.

[4] This SNR m.g.f. was then written an infinite linear combination of m.g.f.s of Gamma distributions in [12, Eq. (37)].

C. Goals and Approach

Herein, we explore and exploit the relationship between the matrix-SC and ZF SNRs to statistically characterize the latter when several streams may experience Rician fading. Also, we aim to reveal the necessary and sufficient *condition* for the matrix-SC to become CWD, and for the virtual Gamma distribution to become the exact distribution of ZF SNRs.

Thus, based on the matrix-matrix partitioning $\mathbf{H} = (\mathbf{H}_1 \quad \mathbf{H}_2)$, where $\mathbf{H}_1$ is $N_{\rm R} \times v$, we characterize the distribution of the ensuing $v \times v$ matrix-SC of $\mathbf{W}_{22}$ in $\mathbf{W}$, denoted $\mathbf{\Gamma}_1$. This helps characterize the ZF SNR distributions of streams corresponding to $\mathbf{H}_1$ when several streams may undergo Rician fading. It also helps reveal the sought *condition*.

D. Contributions

First, for full-Rician fading, we show that $\mathbf{\Gamma}_1$ conditioned on $\mathbf{H}_2$ is NCWD, and state the necessary and sufficient *condition*—found to be a special relationship between the means and column-correlations of $\mathbf{H}_1$ and $\mathbf{H}_2$—that yields a CWD for the unconditioned $\mathbf{\Gamma}_1$, and Gamma distributions for ZF SNRs. Then, we prove that the actual and virtual CWDs for the matrix-SC $\mathbf{\Gamma}_1$ coincide under the *condition*. Consequently, the actual (generally unknown) and virtual (Gamma) distributions of the ZF SNRs for the streams corresponding to $\mathbf{H}_1$ also coincide. Thus, surprisingly, although these streams may undergo Rician fading, their SNRs are distributed as when they undergo Rayleigh fading, which has not been known possible. Importantly, this *condition* qualifies analytically, for the first time, the relationship between the actual distribution of the ZF SNR under Rician fading and the virtual Gamma distribution. Thus, it helps corroborate approximation-accuracy observations from [9], [12, Figs. 1, 2, and 10]. Then, as it is unrelated to condition $r = 1$ imposed in [9, Refs. 24–27, 30, and 31], it also explains the inconsistent approximation accuracy found for $r = 1$ in [9].

Second, we characterize exactly the distribution of the matrix-SC $\mathbf{\Gamma}_1$ for zero-mean $\mathbf{H}_2$, i.e., for Rician(v)/Rayleigh $(N_{\rm T} - v)$ fading. The m.g.f. of $\mathbf{\Gamma}_1$ is deduced in terms of the hypergeometric function ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$, where $\mathbf{S}$ and $\mathbf{\Lambda}$ are $N_{\rm R} \times N_{\rm R}$ matrices. Then, new expressions for ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$ are derived in terms of the determinant of a matrix with elementary-function entries. Specializing to the case of Rician(1)/Rayleigh $(N_{\rm T} - 1)$ fading yields a new determinantal expression for the Stream-1 SNR m.g.f.. Comparing the old [12, Eq. (31)] and new SNR m.g.f. expressions reveals, for when $\mathbf{S}$ has single nonzero eigenvalue σ_1 and $\mathbf{\Lambda}$ is rank-N idempotent, the previously-unknown hypergeometric function relationship ${}_0F_0(\mathbf{S}, \mathbf{\Lambda}) = {}_1F_1(N; N_{\rm R}; \sigma_1)$, and, consequently, a new determinantal expression for the latter.

Finally, numerical results obtained from AEP expressions deduced from the newly-derived exact SNR-m.g.f. expressions reveal the intriguing effects of the mean-correlation *condition* on the relative performance of MIMO ZF for Rician fading vs. Rayleigh-only fading, and of the interference fading type and fading correlation on ZF performance.

E. Notation

Scalars, vectors, and matrices are represented with lowercase italics, lowercase boldface, and uppercase boldface, respectively, e.g., h, $\mathbf{h}$, and $\mathbf{H}$; $\mathbf{h} \sim \mathcal{CN}(\mathbf{h}_\mathrm{d}, \mathbf{R})$ indicates that $\mathbf{h}$ is a complex-valued circularly-symmetric Gaussian random vector [2, p. 39], [22] with mean (i.e., deterministic component) $\mathbf{h}_\mathrm{d}$ and covariance matrix $\mathbf{R}$; $\mathbf{H} \doteq N_\mathrm{R} \times N_\mathrm{T}$ indicates that matrix $\mathbf{H}$ has N_R rows and N_T columns; $\mathbf{H} \sim \mathcal{CN}(\mathbf{H}_\mathrm{d}, \mathbf{I}_{N_\mathrm{R}} \otimes \mathbf{R}_{\mathrm{T},K})$ indicates that $\mathbf{H}$ is complex circularly-symmetric Gaussian matrix with mean $\mathbf{H}_\mathrm{d}$ and transmit-side covariance matrix $\mathbf{R}_{\mathrm{T},K}$ [15]; $r = \mathrm{rank}(\mathbf{H}_\mathrm{d})$; subscripts $\cdot_\mathrm{d}$ and $\cdot_\mathrm{r}$ identify deterministic and random components, respectively; subscript $\cdot_\mathrm{norm}$ indicates a normalized variable; $1:N$ stands for the enumeration $1, 2, \cdots, N$; superscripts $\cdot^\mathcal{T}$ and $\cdot^\mathcal{H}$ stand for transpose and Hermitian (i.e., complex-conjugate) transpose, respectively; $[\mathbf{H}]_{i,j}$ indicates the i,jth (scalar) element of matrix $\mathbf{H}$; $\mathcal{CW}_{N_\mathrm{T}}(N_\mathrm{R}, \mathbf{R}_\mathrm{T})$ denotes the complex CWD with dimension N_T, degrees of freedom N_R, and scale matrix $\mathbf{R}_\mathrm{T}$; $\mathcal{CW}_{N_\mathrm{T}}(N_\mathrm{R}, \mathbf{R}_{\mathrm{T},K}, \mathbf{R}_{\mathrm{T},K}^{-1}\mathbf{H}_\mathrm{d}^\mathcal{H}\mathbf{H}_\mathrm{d})$ denotes the complex NCWD with dimension N_T, degrees of freedom N_R, scale matrix $\mathbf{R}_{\mathrm{T},K}$, and noncentrality parameter matrix $\mathbf{R}_{\mathrm{T},K}^{-1}\mathbf{H}_\mathrm{d}^\mathcal{H}\mathbf{H}_\mathrm{d}$ [15]; $\mathcal{G}(N, \Gamma)$ denotes the Gamma distribution with shape N and scale Γ; $\mathbf{H}_1$ and $\mathbf{H}_2$ are the submatrices obtained by partitioning $\mathbf{H}$ along its columns as in $\mathbf{H} = (\mathbf{H}_1 \quad \mathbf{H}_2)$; accordingly, $\mathbf{W}_{11}$, $\mathbf{W}_{12}$, $\mathbf{W}_{21}$, and $\mathbf{W}_{22}$ are the submatrices obtained by partitioning the Gramian matrix $\mathbf{W} = \mathbf{H}^\mathcal{H}\mathbf{H}$ along rows and columns, so that $\mathbf{W}_{i,j} = \mathbf{H}_i^\mathcal{H}\mathbf{H}_j$, with $i, j = 1, 2$; $\mathbf{W}^{11}$, $\mathbf{W}^{12}$, $\mathbf{W}^{21}$, and $\mathbf{W}^{22}$ are the submatrices obtained by partitioning $\mathbf{W}^{-1}$; the SC of $\mathbf{W}_{22}$ in $\mathbf{W}$ is given by $\mathbf{\Gamma}_1 = (\mathbf{W}^{11})^{-1} = \mathbf{W}_{11} - \mathbf{W}_{12}\mathbf{W}_{22}^{-1}\mathbf{W}_{21}$ [19], [20]; $\mathbf{\Gamma}_1|\mathbf{H}_2$ represents the random matrix $\mathbf{\Gamma}_1$ conditioned on matrix $\mathbf{H}_2$; $\|\mathbf{H}\|^2 = \sum_i^{N_\mathrm{R}}\sum_j^{N_\mathrm{T}} |[\mathbf{H}]_{i,j}|^2 = \mathrm{tr}(\mathbf{H}^\mathcal{H}\mathbf{H})$ is the squared Frobenius norm of $\mathbf{H}$; $\mathrm{tr}(\mathbf{X})$ represents the trace of $\mathbf{X}$, and $\mathrm{etr}(\mathbf{X}) = e^{\mathrm{tr}(\mathbf{X})}$; $\mathbf{0}$ is the zero vector or matrix of appropriate dimensions; $\mathrm{diag}(\cdot, \cdots, \cdot)$ is the diagonal matrix with given elements; $\mathbb{E}\{\cdot\}$ denotes statistical average; $\overset{d}{=}$ and $\overset{d}{\approx}$ relate random variables with the same and approximately the same distribution, respectively; ${}_0F_0(\mathbf{S})$ is the hypergeometric function with a single matrix argument defined in [23, Eq. (35.8.1), p. 772] and characterized by ${}_0F_0(\mathbf{S}) = \mathrm{etr}(\mathbf{S})$ [23, Eq. (35.8.2)]; ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$ is the hypergeometric function of double matrix argument [24, Eq. (88)], [25, Eq. (9)]; ${}_1F_1(\cdot;\cdot;\sigma_1)$ is the confluent hypergeometric function of scalar argument σ_1 [23, Eq. (13.2.2), p. 322]; $(N)_n$ is the Pochhammer symbol, i.e., $(N)_0 = 1$ and $(N)_n = N(N+1)\ldots(N+n-1)$, $\forall n > 1$, [23, p. xiv]; finally, $\Rightarrow$ and $\Leftrightarrow$ represent implication and equivalence, respectively; "iff" is short for "if and only if."

F. Paper Outline

Section II shows our model and details our assumptions. Section III explains the SC–SNR relationship and characterizes the conditioned SC as NCWD. Section IV reveals the mean-correlation *condition* for the SC to be CWD and for ZF SNRs to be Gamma-distributed. Section V shows that the obtained Gamma distribution coincides with the virtual Gamma distribution under the revealed *condition*. Section VI characterizes the matrix-SC distribution for Rician(v)/Rayleigh$(N_\mathrm{T} - v)$ fading. Finally, Section VII presents numerical results.

II. Signal, Channel, and Noise Models

Similarly to [9], [12], this paper considers an uncoded MIMO system over a frequency-flat fading channel. There are N_T and N_R antenna elements at the transmitter(s) and receiver, respectively, with $N_\mathrm{T} \le N_\mathrm{R}$. Letting $\mathbf{x} = [x_1\ x_2\ \cdots\ x_{N_\mathrm{T}}]^\mathcal{T} \doteq N_\mathrm{T} \times 1$ denote the zero-mean transmit-symbol vector with $\mathbb{E}\{\mathbf{x}\mathbf{x}^\mathcal{H}\} = \mathbf{I}_{N_\mathrm{T}}$, the vector with received signals is [2, p. 63]

$$\mathbf{r} = \sqrt{\frac{E_\mathrm{s}}{N_\mathrm{T}}}\,\mathbf{H}\mathbf{x} + \mathbf{n} \doteq\ N_\mathrm{R} \times 1. \tag{1}$$

Above, $E_\mathrm{s}/N_\mathrm{T}$ is the energy transmitted per symbol (i.e., per antenna), so that E_s is the energy transmitted per channel use. The additive noise vector $\mathbf{n}$ is uncorrelated, circularly-symmetric, zero-mean, complex Gaussian with $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\ \mathbf{I}_{N_\mathrm{R}})$ [2, p. 39], [22]; $\widetilde{\mathbf{n}} = \mathbf{n}/\sqrt{N_0} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_\mathrm{R}})$ will also be employed. Then, the per-symbol transmit-SNR is

$$\Gamma_\mathrm{s} = \frac{E_\mathrm{s}}{N_0}\frac{1}{N_\mathrm{T}}. \tag{2}$$

In (1), $\mathbf{H} \doteq N_\mathrm{R} \times N_\mathrm{T}$ is the complex-Gaussian channel matrix, assumed to have rank N_T. Its deterministic and random components are denoted as $\mathbf{H}_\mathrm{d}$ and $\mathbf{H}_\mathrm{r}$, respectively. The channel matrix for Rician fading is usually written as [2, p. 41]

$$\mathbf{H} = \mathbf{H}_\mathrm{d} + \mathbf{H}_\mathrm{r} = \sqrt{\frac{K}{K+1}}\,\mathbf{H}_\mathrm{d,norm} + \sqrt{\frac{1}{K+1}}\,\mathbf{H}_\mathrm{r,norm} \tag{3}$$

where it is assumed, for normalization purposes [26], that:

$$\|\mathbf{H}_\mathrm{d,norm}\|^2 = \mathbb{E}\left\{\|\mathbf{H}_\mathrm{r,norm}\|^2\right\} = N_\mathrm{R}N_\mathrm{T}. \tag{4}$$

Then, if $[\mathbf{H}_\mathrm{d}]_{i,j} = 0$, $|[\mathbf{H}]_{i,j}|$ is Rayleigh-distributed; otherwise, $|[\mathbf{H}]_{i,j}|$ is Rician-distributed [27], and the power ratio

$$K = \frac{\|\mathbf{H}_\mathrm{d}\|^2}{\mathbb{E}\left\{\|\mathbf{H}_\mathrm{r}\|^2\right\}} = \frac{\frac{K}{K+1}\|\mathbf{H}_\mathrm{d,norm}\|^2}{\frac{1}{K+1}\mathbb{E}\left\{\|\mathbf{H}_\mathrm{r,norm}\|^2\right\}} \tag{5}$$

is the Rician K-factor.

For analysis tractability, we assume, as in [8], [14], that the receive-side correlation is zero and that any row $\mathbf{g}_\mathrm{r,norm}^\mathcal{H}$ of $\mathbf{H}_\mathrm{r,norm}$ is distributed as $\mathbf{g}_\mathrm{r,norm} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_\mathrm{T})$, where $\mathbf{R}_\mathrm{T}$ is Hermitian (i.e., $\mathbf{R}_\mathrm{T} = \mathbf{R}_\mathrm{T}^\mathcal{H}$). Considering independent $[\mathbf{H}_\mathrm{r,w,norm}]_{i,j} \sim \mathcal{CN}(0, 1)$, $i = 1:N_\mathrm{R}$, $j = 1:N_\mathrm{T}$, we can write $\mathbf{H}_\mathrm{r,norm} = \mathbf{H}_\mathrm{r,w,norm}\mathbf{R}_\mathrm{T}^{1/2}$, which helps show that $\mathbb{E}\{\|\mathbf{H}_\mathrm{r,norm}\|^2\} = N_\mathrm{R}N_\mathrm{T} \Leftrightarrow \mathrm{tr}(\mathbf{R}_\mathrm{T}) = N_\mathrm{T}$. Therefore, our normalization model (4) allows for unequal $[\mathbf{R}_\mathrm{T}]_{i,i}$, $i = 1:N_\mathrm{T}$, as long as $\sum_{i=1}^{N_\mathrm{T}}[\mathbf{R}_\mathrm{T}]_{i,i} = N_\mathrm{T}$. Thus, the model allows for $\mathbb{E}\{|[\mathbf{H}_\mathrm{r,norm}]_{i,j}|^2\} \neq \mathbb{E}\{|[\mathbf{H}_\mathrm{r,norm}]_{i,k}|^2\}$, $\forall i = 1:N_\mathrm{R}$, $\forall j \neq k$. On the other hand, the elements of $\mathbf{H}_\mathrm{d,norm}$ may have different amplitudes and phases as long as the entire matrix satisfies $\|\mathbf{H}_\mathrm{d,norm}\|^2 = N_\mathrm{R}N_\mathrm{T}$. In conclusion, our analysis applies for both collocated and non-collocated transmit antennas.

Based on the above assumptions, any row $\mathbf{g}_{\mathrm{r}}^{\mathcal{H}}$ of $\mathbf{H}_{\mathrm{r}}$ is characterized by $\mathbf{g}_{\mathrm{r}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathrm{T},K})$, where [9, Eq. (5)]

$$\mathbf{R}_{\mathrm{T},K} = \mathbb{E}\left\{\mathbf{g}_{\mathrm{r}}\mathbf{g}_{\mathrm{r}}^{\mathcal{H}}\right\} = \frac{1}{N_{\mathrm{R}}}\mathbb{E}\left\{\mathbf{H}_{\mathrm{r}}^{\mathcal{H}}\mathbf{H}_{\mathrm{r}}\right\} = \frac{1}{K+1}\mathbf{R}_{\mathrm{T}}, \quad (6)$$

so that $\mathbf{H} \sim \mathcal{CN}(\mathbf{H}_{\mathrm{d}}, \mathbf{I}_{N_{\mathrm{R}}} \otimes \mathbf{R}_{\mathrm{T},K})$ [15].

Matrix $\mathbf{R}_{\mathrm{T}}$ can be computed from the azimuth spread (AS) as shown in [9, Sec. VI.A], when assuming Laplacian power azimuth spectrum, as adopted in WINNER II [11]. Measured AS (in degrees) and K were modeled in WINNER II with scenario-dependent lognormal distributions.

III. ZF SNR Relationship With Schur Complement in Wishart Gramian Matrix

A. Matrix Partitionings and Related Equalities

We introduce below a series of matrix partitionings, decompositions, and ensuing relationships that will be employed throughout. In [12] we employed the vector-matrix partition

$$\mathbf{H} = (\mathbf{h}_1 \quad \mathbf{H}_2) = (\mathbf{h}_{\mathrm{d},1} \quad \mathbf{H}_{\mathrm{d},2}) + (\mathbf{h}_{\mathrm{r},1} \quad \mathbf{H}_{\mathrm{r},2}), \quad (7)$$

where $\mathbf{h}_1$, $\mathbf{h}_{\mathrm{d},1}$, and $\mathbf{h}_{\mathrm{r},1}$ are $N_{\mathrm{R}} \times 1$ vectors, whereas $\mathbf{H}_2$, $\mathbf{H}_{\mathrm{d},2}$, $\mathbf{H}_{\mathrm{r},2}$ are $N_{\mathrm{R}} \times (N_{\mathrm{T}} - 1)$ matrices. However, partitioning (7) can help characterize only the performance for the transmitted stream affected by vector $\mathbf{h}_1$, referred to herein as Stream 1.

Herein, we employ instead the matrix-matrix partitioning

$$\mathbf{H} = (\mathbf{H}_1\ \mathbf{H}_2) = (\mathbf{H}_{\mathrm{d},1}\ \mathbf{H}_{\mathrm{d},2}) + (\mathbf{H}_{\mathrm{r},1}\ \mathbf{H}_{\mathrm{r},2}), \quad (8)$$

where $\mathbf{H}_1$, $\mathbf{H}_{\mathrm{d},1}$, and $\mathbf{H}_{\mathrm{r},1}$ are $N_{\mathrm{R}} \times v$ matrices, whereas $\mathbf{H}_2$, $\mathbf{H}_{\mathrm{d},2}$, $\mathbf{H}_{\mathrm{r},2}$ are $N_{\mathrm{R}} \times (N_{\mathrm{T}} - v)$ matrices, with $v = 1 : N_{\mathrm{T}}$. According to (8), we partition the column-sample-correlation matrix of $\mathbf{H}$, i.e., the Gramian $\mathbf{W} = \mathbf{H}^{\mathcal{H}}\mathbf{H}$, and its inverse $\mathbf{W}^{-1}$ as mentioned in Section I-E. We also partition the covariance matrix $\mathbf{R}_{\mathrm{T},K}$ into its component submatrices $\mathbf{R}_{\mathrm{T},K_{11}}$, $\mathbf{R}_{\mathrm{T},K_{12}}$, $\mathbf{R}_{\mathrm{T},K_{21}}$, and $\mathbf{R}_{\mathrm{T},K_{22}}$, where $\mathbf{R}_{\mathrm{T},K_{21}} = \mathbf{R}_{\mathrm{T},K_{12}}^{\mathcal{H}}$. Also, we partition $\mathbf{R}_{\mathrm{T},K}^{-1}$ into its component submatrices $\mathbf{R}_{\mathrm{T},K}^{11}$, $\mathbf{R}_{\mathrm{T},K}^{12}$, $\mathbf{R}_{\mathrm{T},K}^{21}$, and $\mathbf{R}_{\mathrm{T},K}^{22}$. Further, for $\mathbf{R}_{\mathrm{T},K}$ we consider the upper-lower triangular (UL) decomposition $\mathbf{R}_{\mathrm{T},K} = \mathbf{A}\mathbf{A}^{\mathcal{H}}$ [13, Sec. 5.6], and partition the upper triangular matrix $\mathbf{A}$ into its component submatrices $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21} = \mathbf{0}$, and $\mathbf{A}_{22}$. Finally, we partition $\mathbf{A}^{-1}$ into its component submatrices $\mathbf{A}^{11}$, $\mathbf{A}^{12}$, $\mathbf{A}^{21} = \mathbf{0}$, and $\mathbf{A}^{22}$. For these matrices we have deduced the following relationships, for subsequent use:

$$\mathbf{A}_{11}^{-1} = \mathbf{A}^{11},\ \mathbf{A}_{22}^{-1} = \mathbf{A}^{22}, \mathbf{A}^{12} = -\mathbf{A}^{11}\mathbf{A}_{12}\mathbf{A}^{22} \quad (9)$$

$$\mathbf{R}_{\mathrm{T},K_{22}}^{-1} = (\mathbf{A}_{22}\mathbf{A}_{22}^{\mathcal{H}})^{-1} = \mathbf{A}_{22}^{-\mathcal{H}}\mathbf{A}_{22}^{-1} = \mathbf{A}^{22,\mathcal{H}}\mathbf{A}^{22} \quad (10)$$

$$\mathbf{R}_{\mathrm{T},K_{21}} = \mathbf{A}_{22}\mathbf{A}_{12}^{\mathcal{H}} \quad (11)$$

$$\mathbf{A}_{11}\mathbf{A}_{11}^{\mathcal{H}} = (\mathbf{A}^{11,\mathcal{H}}\mathbf{A}^{11})^{-1} = (\mathbf{R}_{\mathrm{T},K}^{11})^{-1} \quad (12)$$

$$= \mathbf{R}_{\mathrm{T},K_{11}} - \mathbf{R}_{\mathrm{T},K_{12}}\,\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\,\mathbf{R}_{\mathrm{T},K_{21}}. \quad (13)$$

Remark 1: The matrix described by (12) and (13) is referred to as the Schur complement (SC) of $\mathbf{R}_{\mathrm{T},K_{22}}$ in $\mathbf{R}_{\mathrm{T},K}$ [19], [20], [13, Sec. 3.4], [8, Appendix]. For our channel model, it represents the correlation of the first v elements of $\mathbf{g}_{\mathrm{r}}$ given its remaining $N_{\mathrm{T}} - v$ elements.

B. Schur Complement in the Gramian Matrix $\mathbf{W}$

The SC of $\mathbf{W}_{22} = \mathbf{H}_2^{\mathcal{H}}\mathbf{H}_2$ in Gramian $\mathbf{W}$ is the matrix

$$\mathbf{\Gamma}_1 = (\mathbf{W}^{11})^{-1} = \mathbf{W}_{11} - \mathbf{W}_{12}\mathbf{W}_{22}^{-1}\mathbf{W}_{21} \doteq v \times v. \quad (14)$$

It can be expressed as a matrix Hermitian form as follows:

$$\mathbf{\Gamma}_1 = \mathbf{H}_1^{\mathcal{H}}\mathbf{H}_1 - \mathbf{H}_1^{\mathcal{H}}\mathbf{H}_2(\mathbf{H}_2^{\mathcal{H}}\mathbf{H}_2)^{-1}\mathbf{H}_2^{\mathcal{H}}\mathbf{H}_1 \quad (15)$$

$$= \mathbf{H}_1^{\mathcal{H}}\underbrace{\left[\mathbf{I}_{N_{\mathrm{R}}} - \mathbf{H}_2\left(\mathbf{H}_2^{\mathcal{H}}\mathbf{H}_2\right)^{-1}\mathbf{H}_2^{\mathcal{H}}\right]}_{=\mathbf{Q}_2}\mathbf{H}_1. \quad (16)$$

First, note from (15) that the SC matrix $\mathbf{\Gamma}_1$ is the column sample-correlation of $\mathbf{H}_1$ given $\mathbf{H}_2$. Then, note that matrix $\mathbf{H}_2(\mathbf{H}_2^{\mathcal{H}}\mathbf{H}_2)^{-1}\mathbf{H}_2^{\mathcal{H}} \doteq N_{\mathrm{R}} \times N_{\mathrm{R}}$ is the projection onto the column space of $\mathbf{H}_2$, whereas matrix $\mathbf{Q}_2 \doteq N_{\mathrm{R}} \times N_{\mathrm{R}}$ is the projection onto the null space of $\mathbf{H}_2^{\mathcal{H}}$. These Hermitian matrices are idempotent and have eigenvalues as listed below:

$$\mathbf{H}_2\left(\mathbf{H}_2^{\mathcal{H}}\mathbf{H}_2\right)^{-1}\mathbf{H}_2^{\mathcal{H}} :\ 1,\ 1,\ \cdots,\ 1, \quad 0,\ 0,\ \cdots,\ 0 \quad (17)$$

$$\mathbf{Q}_2 :\ \underbrace{0,\ 0,\ \cdots,\ 0,}_{N_{\mathrm{T}}-v} \quad \underbrace{1,\ 1,\ \cdots,\ 1}_{N_{\mathrm{R}}-N_{\mathrm{T}}+v=N_v} \quad (18)$$

Their ranks are $N_{\mathrm{T}} - v$ and N_v, respectively. We shall denote N_v for $v = 1$, i.e., $N_{\mathrm{R}} - N_{\mathrm{T}} + 1$, simply as N, as in [12].

C. Relationship of SC $\mathbf{\Gamma}_1$ With ZF SNRs

Given $\mathbf{H}$ and nonsingular $\mathbf{W} = \mathbf{H}^{\mathcal{H}}\mathbf{H}$, ZF for the signal from (1) means separately mapping into the closest modulation (e.g., MPSK) constellation symbol each element of the following $N_{\mathrm{T}} \times 1$ vector [2, p. 153]:

$$\mathbf{y} = \sqrt{\frac{N_{\mathrm{T}}}{E_{\mathrm{s}}}}[\mathbf{H}^{\mathcal{H}}\mathbf{H}]^{-1}\mathbf{H}^{\mathcal{H}}\,\mathbf{r} = \mathbf{x} + \frac{1}{\sqrt{\Gamma_{\mathrm{s}}}}[\mathbf{H}^{\mathcal{H}}\mathbf{H}]^{-1}\mathbf{H}^{\mathcal{H}}\tilde{\mathbf{n}}. \quad (19)$$

Since the resulting noise vector has correlation matrix $\mathbf{W}^{-1}/\Gamma_{\mathrm{s}}$, the ZF SNR for Stream $i = 1 : N_{\mathrm{T}}$ has usually been expressed in ratio form as follows [2, p. 153], [14]

$$\gamma_i = \frac{\Gamma_{\mathrm{s}}}{[\mathbf{W}^{-1}]_{i,i}}. \quad (20)$$

Now, $\forall\, v = 1 : N_{\mathrm{T}}$ we can write, based on (14), that

$$\gamma_i = \frac{\Gamma_{\mathrm{s}}}{[\mathbf{W}^{-1}]_{i,i}} = \frac{\Gamma_{\mathrm{s}}}{[\mathbf{W}^{11}]_{i,i}} = \frac{\Gamma_{\mathrm{s}}}{\left[\mathbf{\Gamma}_1^{-1}\right]_{i,i}}, \quad i = 1 : v. \quad (21)$$

Thus, in general $(\forall\, v)$, the ZF SNRs for Streams $i = 1 : v$ are determined by the SC matrix $\mathbf{\Gamma}_1$, through its inverse $\mathbf{\Gamma}_1^{-1}$. Only for $v = 1$, i.e., when $\mathbf{\Gamma}_1$ reduces to a scalar, we can write the ZF SNR for Stream 1 in terms of the SC as [12]

$$\gamma_1 = \frac{\Gamma_{\mathrm{s}}}{[\mathbf{W}^{-1}]_{1,1}} = \frac{\Gamma_{\mathrm{s}}}{\mathbf{W}^{11}} = \Gamma_{\mathrm{s}}(\mathbf{W}^{11})^{-1} = \Gamma_{\mathrm{s}}\mathbf{\Gamma}_1. \quad (22)$$

Based on (15), we can put the SC in scalar Hermitian form

$$\boldsymbol{\Gamma}_1 = \mathbf{h}_1^{\mathcal{H}} \mathbf{Q}_2 \mathbf{h}_1, \tag{23}$$

which has helped characterize the distribution of γ_1 for Rician(1)/Rayleigh $(N_{\rm T}-1)$ fading as an infinite linear combination of Gamma distributions in [12, Eq. (37)] — see also (59) and (60).

For the more general case $v \geq 1$, we analyze hereafter in this paper the distribution of the matrix-SC $\boldsymbol{\Gamma}_1$ based on its Hermitian form (16) and exploit its relationship from (21) with ZF SNRs to analyze the ZF performance for Streams $i = 1 : v$.

D. Distribution of SC $\boldsymbol{\Gamma}_1$ Conditioned on $\mathbf{H}_2$ (or $\mathbf{Q}_2$)

Nonzero- and zero-mean complex-Gaussian $\mathbf{H}$ yield complex NCWD and CWD Gramian $\mathbf{W}$, respectively [15]:

$$\begin{aligned} &\mathbf{H} \sim \mathcal{CN}\left(\mathbf{H}_{\rm d}, \mathbf{I}_{N_{\rm R}} \otimes \mathbf{R}_{{\rm T},K}\right) \\ &\qquad \Rightarrow \mathbf{W} \sim \mathcal{CW}_{N_{\rm T}}\left(N_{\rm R}, \mathbf{R}_{{\rm T},K}, \mathbf{R}_{{\rm T},K}^{-1}\mathbf{H}_{\rm d}^{\mathcal{H}}\mathbf{H}_{\rm d}\right) \end{aligned} \tag{24}$$

$$\begin{aligned} &\mathbf{H} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{N_{\rm R}} \otimes \mathbf{R}_{\rm T}\right) \\ &\qquad \Rightarrow \mathbf{W} \sim \mathcal{CW}_{N_{\rm T}}(N_{\rm R}, \mathbf{R}_{\rm T}). \end{aligned} \tag{25}$$

Because $\mathbf{H}_1$ and $\mathbf{H}_2$ are jointly Gaussian, the distribution of $\mathbf{H}_1$ given $\mathbf{H}_2$ is [8, Appendix]

$$\mathbf{H}_1|\mathbf{H}_2 \sim \mathcal{CN}\left(\mathbf{M} + \mathbf{H}_2\mathbf{R}_{2,1}, \mathbf{I}_{N_{\rm R}} \otimes \left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right), \tag{26}$$

[5]where

$$\mathbf{M} = \mathbf{H}_{{\rm d},1} - \mathbf{H}_{{\rm d},2}\mathbf{R}_{2,1} \doteq N_{\rm R} \times v, \tag{27}$$

$$\mathbf{R}_{2,1} = \mathbf{R}_{{\rm T},K_{22}}^{-1}\mathbf{R}_{{\rm T},K_{21}} \doteq (N_{\rm T} - v) \times v, \tag{28}$$

are deterministic matrices. We can now recast (26) further as

$$\mathbf{H}_1|\mathbf{H}_2 \overset{d}{=} \mathbf{X} + \mathbf{H}_2\mathbf{R}_{2,1}; \mathbf{X} \sim \mathcal{CN}\left(\mathbf{M}, \mathbf{I}_{N_{\rm R}} \otimes \left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right). \tag{29}$$

Substituting this in (16) and manipulating as in [8] yields

$$\boldsymbol{\Gamma}_1|\mathbf{Q}_2 \overset{d}{=} \mathbf{X}^{\mathcal{H}}\mathbf{Q}_2\mathbf{X}. \tag{30}$$

This matrix Hermitian form has, for $\mathbf{M} \neq \mathbf{0}$, the NCWD [28, Cor. 7.8.1.1, p. 255]

$$\boldsymbol{\Gamma}_1|\mathbf{Q}_2 \sim \mathcal{CW}_v\left(N_v, \left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}, \mathbf{R}_{{\rm T},K}^{11}\mathbf{M}^{\mathcal{H}}\mathbf{Q}_2\mathbf{M}\right). \tag{31}$$

Thus, its m.g.f. for matrix $\boldsymbol{\Theta} \doteq v \times v$ is given by [17, Eq. (4)]

$$\begin{aligned} M_{\boldsymbol{\Gamma}_1|\mathbf{Q}_2}(\boldsymbol{\Theta}) &= \left|\mathbf{I}_v - \boldsymbol{\Theta}\left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right|^{-N_v} \\ &\quad \times \operatorname{etr}\left(\left[\mathbf{I}_v - \boldsymbol{\Theta}\left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right]^{-1}\boldsymbol{\Theta}\mathbf{M}^{\mathcal{H}}\mathbf{Q}_2\mathbf{M}\right). \end{aligned} \tag{32}$$

[5]This corroborates Remark 1 on the meaning of SC $(\mathbf{R}_{{\rm T},K}^{11})^{-1}$.

Deriving from (32) the m.g.f. of the unconditioned $\boldsymbol{\Gamma}_1$ as

$$\begin{aligned} M_{\boldsymbol{\Gamma}_1}(\boldsymbol{\Theta}) &= \left|\mathbf{I}_v - \boldsymbol{\Theta}\left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right|^{-N_v} \\ &\quad \times \mathbb{E}_{\mathbf{Q}_2}\left\{\operatorname{etr}\left(\left[\mathbf{I}_v - \boldsymbol{\Theta}\left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right]^{-1}\boldsymbol{\Theta}\mathbf{M}^{\mathcal{H}}\mathbf{Q}_2\mathbf{M}\right)\right\} \end{aligned} \tag{33}$$

remains intractable for general Rician fading. Nevertheless, (33) yields the distribution of the unconditioned $\boldsymbol{\Gamma}_1$ for the following special cases:

1) Full-Rician fading under condition $\mathbf{M} = \mathbf{0}$. Note that $\mathbf{M} = \mathbf{0}$ covers the trivial case of Rayleigh-only fading as well as a case that may be practically relevant: Rayleigh(v)/Rician$(N_{\rm T} - v)$ fading whereby the Rayleigh fading is uncorrelated with the Rician fading.
2) Rician(v)/Rayleigh$(N_{\rm T} - v)$ fading.

They are treated in Sections IV and VI, respectively.

IV. Distribution of ZF SNRs for Streams $i = 1 : v$, $v = 1 : N_{\rm T}$, Under Rician Fading With $\mathbf{M} = \mathbf{0}$

A. $\boldsymbol{\Gamma}_1$ is CWD If and Only If $\mathbf{M} = \mathbf{0}$, i.e., $\mathbf{H}_{{\rm d},1} = \mathbf{H}_{{\rm d},2}\mathbf{R}_{2,1}$

The theorem below follows readily from the fact that in (33) $\mathbb{E}_{\mathbf{Q}_2}\{\operatorname{etr}(\cdot)\} = 1$ *iff* $\mathbf{M} = \mathbf{H}_{{\rm d},1} - \mathbf{H}_{{\rm d},2}\mathbf{R}_{2,1} = \mathbf{0}$.

Theorem 1:

$$\mathbf{H}_{{\rm d},1} = \mathbf{H}_{{\rm d},2}\mathbf{R}_{2,1} \tag{34}$$

$$\Leftrightarrow\ M_{\boldsymbol{\Gamma}_1}(\boldsymbol{\Theta}) = \left|\mathbf{I}_v - \boldsymbol{\Theta}\left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right|^{-N_v} \tag{35}$$

$$\Leftrightarrow\ \boldsymbol{\Gamma}_1 \sim \mathcal{CW}_v\left(N_v, \left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right). \tag{36}$$

Remark 2: The mean-correlation condition (34) holds for:

- Rayleigh-only fading, i.e., $\mathbf{H}_{{\rm d},1} = \mathbf{0}$ and $\mathbf{H}_{{\rm d},2} = \mathbf{0}$ (then, the value of v is irrelevant).
- Rayleigh(v)/Rician$(N_{\rm T} - v)$ fading, i.e., for $\mathbf{H}_{{\rm d},1} = \mathbf{0}$, $\mathbf{H}_{{\rm d},2} \neq \mathbf{0}$, if the Rayleigh fading is uncorrelated with the Rician fading, which reduces to zero $\mathbf{R}_{{\rm T},K_{21}}$, i.e., $\mathbf{R}_{2,1}$, in (28).

For full-Rician fading, condition (34) implies an interesting "parallelism" between means and correlations, as shown in Appendix I, Corollary 8. That Appendix provides some additional analysis and insights into condition (34).

B. ZF SNR Distribution for Streams $i = 1 : v$

For CWD $\boldsymbol{\Gamma}_1$, i.e., for $\mathbf{H}_{{\rm d},1} = \mathbf{H}_{{\rm d},2}\mathbf{R}_{2,1}$, the following Lemma characterizes as Gamma-distributed the ZF SNRs for Streams $i = 1 : v$.

Lemma 1:

$$\boldsymbol{\Gamma}_1 \sim \mathcal{CW}_v\left(N_v, \left(\mathbf{R}_{{\rm T},K}^{11}\right)^{-1}\right) \Rightarrow \text{for } i = 1 : v$$

$$\gamma_i = \frac{\Gamma_{\rm s}}{\left[\boldsymbol{\Gamma}_1^{-1}\right]_{i,i}} \sim \mathcal{G}\left(N, \Gamma_{K,i} = \frac{\Gamma_{\rm s}}{\left[\mathbf{R}_{{\rm T},K}^{-1}\right]_{i,i}}\right). \tag{37}$$

TABLE I
DEPENDENCE OF ZF SNR DISTRIBUTION AND AEP ON FADING TYPE AND MEAN-CORRELATION CONDITION $\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}$ FROM (34)

	Fading Type	$\mathbf{H}_{\mathrm{d},1}$	$\mathbf{H}_{\mathrm{d},2}$	$\mathbf{R}_{2,1}$	(34)	γ_i Distribution, $i = 1 : v$	AEP
1	Rayleigh-only	$= \mathbf{0}$	$= \mathbf{0}$	$\forall$	✓	$\gamma_i \overset{d}{=} \widehat{\gamma}_i \sim \mathcal{G}(N, \Gamma_{0,i})$, see (40)	$P_{\mathrm{e},i}^{(44)} = \widehat{P}_{\mathrm{e},i}^{(50)}$
2	Rayleigh(v)/Rice($N_{\mathrm{T}} - v$)	$= \mathbf{0}$	$\neq \mathbf{0}$	$= \mathbf{0}$	✓	$\gamma_i \overset{d}{=} \widehat{\gamma}_i \sim \mathcal{G}(N, \Gamma_{K,i})$, see (37)	$P_{\mathrm{e},i}^{(44)} = \widehat{P}_{\mathrm{e},i}^{(50)}$
3	Rice(v)/Rice($N_{\mathrm{T}} - v$)	$\neq \mathbf{0}$	$\neq \mathbf{0}$	$\neq \mathbf{0}$	✓	$\gamma_i \overset{d}{=} \widehat{\gamma}_i \sim \mathcal{G}(N, \Gamma_{K,i})$, see (37)	$P_{\mathrm{e},i}^{(44)} = \widehat{P}_{\mathrm{e},i}^{(50)}$
4	Rice(1)/Rayleigh($N_{\mathrm{T}} - 1$)	$\neq \mathbf{0}$	$= \mathbf{0}$	$\forall$	✗	Known for γ_1, see (59), (69)	$P_{\mathrm{e},1}^{(70)}$
5	Rice(v)/Rayleigh($N_{\mathrm{T}} - v$), $v > 1$	$\neq \mathbf{0}$	$= \mathbf{0}$	$\forall$	✗	Unknown; $M_{\mathbf{\Gamma}_1}(\mathbf{\Theta})$ in (66)	Unknown
6	Rayleigh(v)/Rice($N_{\mathrm{T}} - v$)	$= \mathbf{0}$	$\neq \mathbf{0}$	$\neq \mathbf{0}$	✗	Unknown	Unknown
7	Rice(v)/Rice($N_{\mathrm{T}} - v$)	$\neq \mathbf{0}$	$\neq \mathbf{0}$	$\forall$	✗	Unknown	Unknown
8	Virtual Rayleigh	$\neq \mathbf{0}$	$\neq \mathbf{0}$	$\forall$		$\widehat{\gamma}_i \sim \mathcal{G}(N, \widehat{\Gamma}_{K,i})$, $i = 1 : N_{\mathrm{T}}$, see (47)	$\widehat{P}_{\mathrm{e},i}^{(50)}$

Proof: A special case of [16, Th. 3.2.11, p. 95] yields

$$\mathbf{\Gamma}_1 \sim \mathcal{CW}_v\left(N_v, \left(\mathbf{R}_{\mathrm{T},K}^{11}\right)^{-1}\right) \Rightarrow \text{ for } i = 1 : v$$
$$\frac{1}{\left[\mathbf{\Gamma}_1^{-1}\right]_{i,i}} \sim \mathcal{CW}_1\left(N, \frac{1}{\left[\mathbf{R}_{\mathrm{T},K}^{11}\right]_{i,i}}\right).$$

Because for $i = 1 : v$ we can write $[\mathbf{R}_{\mathrm{T},K}^{11}]_{i,i} = [\mathbf{R}_{\mathrm{T},K}^{-1}]_{i,i}$, we can express the m.g.f. of $1/[\mathbf{\Gamma}_1^{-1}]_{i,i}$ as [17, Eq. (4)]

$$M(s) = \left(1 - s/\left[\mathbf{R}_{\mathrm{T},K}^{-1}\right]_{i,i}\right)^{-N}, \tag{38}$$

which yields the desired result, i.e.,

$$M_{\gamma_i}(s) = M(s\Gamma_{\mathrm{s}}) = (1 - s\Gamma_{K,i})^{-N}. \tag{39}$$

Corollary 1: For Rayleigh-only fading $(K = 0)$, the ZF SNRs on all streams $i = 1 : N_{\mathrm{T}}$ are Gamma-distributed as:

$$\gamma_i = \frac{\Gamma_{\mathrm{s}}}{\left[\mathbf{\Gamma}_1^{-1}\right]_{i,i}} \sim \mathcal{G}\left(N, \Gamma_{0,i} = \frac{\Gamma_{\mathrm{s}}}{\left[\mathbf{R}_{\mathrm{T}}^{-1}\right]_{i,i}}\right), \tag{40}$$

whereas for Rician fading $(K \neq 0)$ satisfying $\mathbf{M} = \mathbf{0}$, Streams $i = 1 : v$ are Gamma-distributed as in (37), with[6]

$$\Gamma_{K,i} = \frac{\Gamma_{\mathrm{s}}}{\left[\mathbf{R}_{\mathrm{T},K}^{-1}\right]_{i,i}} = \frac{1}{K+1}\frac{\Gamma_{\mathrm{s}}}{\left[\mathbf{R}_{\mathrm{T}}^{-1}\right]_{i,i}} = \frac{1}{K+1}\Gamma_{0,i}. \tag{41}$$

Remark 3: If $\mathbf{M} = \mathbf{0}$ then Rician fading on any stream

- does not change SNR distribution type (Gamma) for Streams $i = 1 : v$, compared to Rayleigh-only fading; these SNR distributions are also independent of v, $\mathbf{H}_{\mathrm{d}}$;
- reduces the average SNR $(\mathbb{E}\{\gamma_i\} = N\Gamma_{K,i})$ for Streams $i = 1 : v$ by a factor of $K + 1$ over Rayleigh-only fading; this is illustrated numerically in Section VII;
- leaves intractable the derivation of the ZF SNR distributions for streams $i = v + 1 : N_{\mathrm{T}}$.

C. AEP Expression for Streams $i = 1 : v$

Knowing the SNR m.g.f., the elegant AEP-derivation procedure from [27, Ch. 9] can be employed, e.g., for MPSK modulation. Because then the Stream-i error probability is given by [27, Eq. (8.22)]

$$P_{\mathrm{e}}(\gamma_i) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \exp\left\{-\gamma_i \frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\right\} d\theta, \tag{42}$$

the AEP can be written as [27, Ch. 9]

$$P_{\mathrm{e},i} = \mathbb{E}\left\{P_{\mathrm{e}}(\gamma_i)\right\} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} M_{\gamma_i}\left(-\frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\right) d\theta. \tag{43}$$

Substituting the m.g.f. from (39) into (43) yields the *exact AEP expression* for Streams $i = 1 : v$, under condition (34),

$$P_{\mathrm{e},i}^{(44)} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(1 + \frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\Gamma_{K,i}\right)^{-N} d\theta. \tag{44}$$

D. Summary of Results

In Table I, Rows 1–3 characterize, based on Lemma 1 and Corollary 1, ZF SNR distributions for fading cases whereby the mean–correlation condition (34) holds (✓).

Remaining rows characterize fading cases whereby (34) does not hold (✗). Of them, only for the case of Rician(1)/Rayleigh $(N_{\mathrm{T}} - 1)$ fading, characterized in Row 4, we have recently found in [12], by partitioning with $v = 1$, that the exact distribution of γ_1 is an infinite linear combination of Gamma distributions [12, Eq. (37)] — see also (59), (60).

In Section VI, we shall generalize the approach from [12] to the partitioning with $v > 1$, to express the m.g.f. of $\mathbf{\Gamma}_1$ under Rician(v)/Rayleigh$(N_{\mathrm{T}} - v)$ fading, which yields the determinantal expression for the m.g.f. of γ_1 in (69), i.e., an alternative to the infinite-series expression [12, Eq. (37)].

However, below, we first take a fresh look at a Wishart-distribution approximation[7] proposed in [18] and applied for ZF analysis in [9, Refs. 24–27, 30, and 31], without accuracy testing. Our numerical testing from [9] of this approximation revealed only that lower values of $r = \mathrm{rank}(\mathbf{H}_{\mathrm{d}})$ yield — inconsistently — lower ZF SNR distribution-approximation error. Next, we reconsider this approximation analytically and reveal that it turns exact under condition (34).

[6]Based on (6).

[7]Characterized, for convenience, in Row 8 of Table I.

V. APPROXIMATE AND EXACT GAMMA DISTRIBUTIONS FOR ZF SNRS

A. Approximate CWD for $\mathbf{W}$ Proposed in [18]

On one hand, given the actual nonzero-mean channel matrix $\mathbf{H} \sim \mathcal{CN}(\mathbf{H}_\mathrm{d}, \mathbf{I}_{N_\mathrm{R}} \otimes \mathbf{R}_{\mathrm{T},K})$, we have $\mathbf{W} = \mathbf{H}^\mathcal{H}\mathbf{H} \sim \mathcal{CW}_{N_\mathrm{T}}(N_\mathrm{R}, \mathbf{R}_{\mathrm{T},K}, \mathbf{R}_{\mathrm{T},K}^{-1}\mathbf{H}_\mathrm{d}^\mathcal{H}\mathbf{H}_\mathrm{d})$. On the other hand, as in [18], if we consider a virtual zero-mean matrix $\widehat{\mathbf{H}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_\mathrm{R}} \otimes \widehat{\mathbf{R}}_{\mathrm{T},K})$, then $\widehat{\mathbf{W}} = \widehat{\mathbf{H}}^\mathcal{H}\widehat{\mathbf{H}} \sim \mathcal{CW}_{N_\mathrm{T}}(N_\mathrm{R}, \widehat{\mathbf{R}}_{\mathrm{T},K})$. The proof of next Lemma follows using

$$\mathbb{E}\{\mathbf{W}\} = N_\mathrm{R}\mathbf{R}_{\mathrm{T},K} + \mathbf{H}_\mathrm{d}^\mathcal{H}\mathbf{H}_\mathrm{d}, \quad \mathbb{E}\{\widehat{\mathbf{W}}\} = N_\mathrm{R}\widehat{\mathbf{R}}_{\mathrm{T},K}. \tag{45}$$

Lemma 2 ([9], [18]):

$$\mathbb{E}\{\widehat{\mathbf{W}}\} = \mathbb{E}\{\mathbf{W}\} \Leftrightarrow \widehat{\mathbf{R}}_{\mathrm{T},K} = \mathbf{R}_{\mathrm{T},K} + \frac{1}{N_\mathrm{R}}\mathbf{H}_\mathrm{d}^\mathcal{H}\mathbf{H}_\mathrm{d}, \tag{46}$$

i.e., the two Wishart distributions have equal means iff relationship (46) holds between the statistics of $\mathbf{H}$ and $\widehat{\mathbf{H}}$.

Based on the mean-equality (46), the approximation of the NCWD of the actual $\mathbf{W}$ with the virtual CWD of $\widehat{\mathbf{W}}$ was proposed in [18], and was applied for ZF SNR analysis in [9, Refs. 24–27, 30, and 31], as shown next.

B. Ensuing Approximate Gamma Distributions for ZF SNRs Used in [9, Refs. 24–27, 30 and 31] for $r = \mathit{rank}(\mathbf{H}_\mathrm{d}) = 1$

Based on (40), we can write for all the virtual ZF SNRs

$$\widehat{\gamma}_i = \frac{\Gamma_\mathrm{s}}{[\widehat{\mathbf{W}}^{-1}]_{i,i}} \sim \mathcal{G}(N, \widehat{\Gamma}_{K,i}), \widehat{\Gamma}_{K,i} = \frac{\Gamma_\mathrm{s}}{\left[\widehat{\mathbf{R}}_{\mathrm{T},K}^{-1}\right]_{i,i}}, \tag{47}$$

$$M_{\widehat{\gamma}_i}(s) = (1 - s\widehat{\Gamma}_{K,i})^{-N}, \quad i = 1 : N_\mathrm{T}. \tag{48}$$

Then, the approximation in distribution $\mathbf{W} \overset{d}{\approx} \widehat{\mathbf{W}}$ from [18] led in [9, Refs. 24–27, 30, and 31] to the approximation in distribution

$$\gamma_i \overset{d}{\approx} \widehat{\gamma}_i \sim \mathcal{G}(N, \widehat{\Gamma}_{K,i}), \quad i = 1 : N_\mathrm{T}. \tag{49}$$

Finally, substituting the m.g.f. from (48) into (43) has yielded the *approximate AEP expression* [9, Eq. (39)]

$$\widehat{P}_{\mathrm{e},i}^{(50)} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi}\left(1 + \frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\widehat{\Gamma}_{K,i}\right)^{-N} d\theta, \quad i = 1 \colon N_\mathrm{T}. \tag{50}$$

The virtual SNR distribution (47) and the ensuing $\widehat{P}_{\mathrm{e},i}^{(50)}$ are referenced on Row 8 in Table I, for virtual Rayleigh fading.

C. Analogous Approximate CWD for SC $\mathbf{\Gamma}_1$

Given $v = 1 : N_\mathrm{T}$, let us partition $\widehat{\mathbf{H}}$, $\widehat{\mathbf{R}}_{\mathrm{T},K}$, $\widehat{\mathbf{R}}_{\mathrm{T},K}^{-1}$, $\widehat{\mathbf{W}}$, and $\widehat{\mathbf{W}}^{-1}$ as done for $\mathbf{H}$, $\mathbf{R}_{\mathrm{T},K}$, $\mathbf{R}_{\mathrm{T},K}^{-1}$, $\mathbf{W}$, and $\mathbf{W}^{-1}$ in Section II. Also, analogously to the actual SC $\mathbf{\Gamma}_1$ defined in (14), let us define the virtual SC

$$\widehat{\mathbf{\Gamma}}_1 = (\widehat{\mathbf{W}}^{11})^{-1} = \widehat{\mathbf{W}}_{11} - \widehat{\mathbf{W}}_{12}\widehat{\mathbf{W}}_{22}^{-1}\widehat{\mathbf{W}}_{21}. \tag{51}$$

Then, analogously to (21), we can write

$$\widehat{\gamma}_i = \frac{\Gamma_\mathrm{s}}{[\widehat{\mathbf{W}}^{-1}]_{i,i}} = \frac{\Gamma_\mathrm{s}}{\left[\widehat{\mathbf{\Gamma}}_1^{-1}\right]_{i,i}}, \quad i = 1 : N_\mathrm{T}. \tag{52}$$

Since $\widehat{\mathbf{H}}$ is zero-mean, $\widehat{\mathbf{\Gamma}}_1$ has the m.g.f.

$$M_{\widehat{\mathbf{\Gamma}}_1}(\mathbf{\Theta}) = \left|\mathbf{I}_v - \mathbf{\Theta}\left(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11}\right)^{-1}\right|^{-N_v}, \tag{53}$$

i.e., matrix $\widehat{\mathbf{\Gamma}}_1$ has the following CWD:

$$\widehat{\mathbf{\Gamma}}_1 \sim \mathcal{CW}_v\left(N_v, \left(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11}\right)^{-1}\right). \tag{54}$$

Based on the approximation in distribution $\mathbf{W} \overset{d}{\approx} \widehat{\mathbf{W}}$ proposed in [18], one may view $\widehat{\mathbf{\Gamma}}_1$ as approximating $\mathbf{\Gamma}_1$ in distribution. This view is also supported by the fact that the generally-unknown distribution of $\mathbf{\Gamma}_1$ and the CWD of $\widehat{\mathbf{\Gamma}}_1$ turn exactly the same under condition (34), as shown next.

D. Condition for $\mathbf{\Gamma}_1 \overset{d}{=} \widehat{\mathbf{\Gamma}}_1$, and for $\gamma_i \overset{d}{=} \widehat{\gamma}_i$, $i = 1 : v$

Theorem 2:

$$\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1} \Leftrightarrow \left(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11}\right)^{-1} = \left(\mathbf{R}_{\mathrm{T},K}^{11}\right)^{-1}. \tag{55}$$

Proof: See Appendix II.

Corollary 2: Theorems 1 and 2, along with (54), yield:

$$\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1} \Leftrightarrow \widehat{\mathbf{\Gamma}}_1 \overset{d}{=} \mathbf{\Gamma}_1 \sim \mathcal{CW}_v\left(N_v, \left(\mathbf{R}_{\mathrm{T},K}^{11}\right)^{-1}\right). \tag{56}$$

Corollary 3: The SNR–SC relationships from (21) and (52) along with equivalence (56) yield the implication

$$\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1} \Rightarrow \forall\, i = 1 : v, \forall\, v = 1 : N_\mathrm{T}$$
$$\gamma_i = \frac{\Gamma_\mathrm{s}}{\left[\mathbf{\Gamma}_1^{-1}\right]_{i,i}} \overset{d}{=} \widehat{\gamma}_i = \frac{\Gamma_\mathrm{s}}{\left[\widehat{\mathbf{\Gamma}}_1^{-1}\right]_{i,i}} \sim \mathcal{G}(N, \Gamma_{K,i}). \tag{57}$$

Note that (57) implies the AEP equality $P_{\mathrm{e},i}^{(44)} = \widehat{P}_{\mathrm{e},i}^{(50)}$, which is depicted in Rows 1–3 of Table I.

Corollary 4: For $v = 1$, i.e., scalar $\mathbf{\Gamma}_1$ and $\widehat{\mathbf{\Gamma}}_1$, equivalence (56) yields equivalence[8]

$$\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1} \Leftrightarrow \gamma_1 \overset{d}{=} \widehat{\gamma}_1 \sim \mathcal{G}(N, \Gamma_{K,1}). \tag{58}$$

E. Corroboration and Explanation of Previous Observations

The equivalence in (58) helps explain our earlier observations that the accuracy of $\gamma_1 \overset{d}{\approx} \widehat{\gamma}_1$ and $P_{\mathrm{e},1}^{(44)} \approx \widehat{P}_{\mathrm{e},1}^{(50)}$ is:

- dependent on the combination of $\mathbf{H}_\mathrm{d}$ and $\mathbf{R}_{\mathrm{T},K}$ [9, Sec. VI.B–E].
- poor for $\mathbf{H}_\mathrm{d}$ with rank $r = N_\mathrm{T}$ [9, Figs. 1 and 2] and for $\mathbf{H}_\mathrm{d} = (\mathbf{h}_{\mathrm{d},1}\ \ \mathbf{0})$ [12, Figs. 1 and 2], whereby $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$ does not hold.

[8]Matrix $\mathbf{R}_{2,1} \doteq (N_\mathrm{T} - v) \times v$ reduces to vector $\mathbf{r}_{2,1} \doteq (N_\mathrm{T} - 1) \times 1$.

- good in [12, Fig. 10] (and Fig. 3 herein) whereby $\mathbf{h}_{\mathrm{d},1} \approx \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$.
- inconsistent for $r = 1$ [9, Sec. VI.C], which is because $r = 1$ and $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$ are unrelated; thus, previous usage for $r = 1$ of $\gamma_1 \overset{d}{\approx} \widehat{\gamma}_1$ in [9, Refs. 24–27, 30, and 31] appears unwarranted.

VI. M.G.F. of Matrix-SC $\mathbf{\Gamma}_1$ Under Rician(v)/Rayleigh$(N_\mathrm{T} - v)$ Fading, $v = 1 : N_\mathrm{T} - 1$

A. $M_{\mathbf{\Gamma}_1}(\mathbf{\Theta})$ *for* $\mathbf{H}_{\mathrm{d},2} = \mathbf{0}$*, in Terms of* ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$

Our recent analysis of the scalar-SC (i.e., for $v = 1$) from [12] yielded for ZF under Rician(1)/Rayleigh $(N_\mathrm{T} - 1)$ fading the SNR m.g.f. for the (Rician) Stream 1 as [12, Eq. (31)]

$$M_{\gamma_1}(s) = (1 - s\Gamma_{K,1})^{-N}\ {}_1F_1(N; N_\mathrm{R}; \sigma_1), \tag{59}$$

with σ_1 shown herein in Appendix III-D, Eq. (94). By substituting the confluent hypergeometric function of scalar argument from its infinite-series expansion around the origin [23, Eq. (13.2.2), p. 322]

$${}_1F_1(N; N_\mathrm{R}; \sigma_1) = \sum_{n=0}^{\infty} \frac{(N)_n}{(N_\mathrm{R})_n} \frac{\sigma_1^n}{n!} \tag{60}$$

into (59), we showed in [12, Eq. (37)] that $M_{\gamma_1}(s)$ is an infinite linear combination of m.g.f.s of Gamma distributions.

Hereafter, a matrix-SC analysis applicable $\forall\, v = 1 : N_\mathrm{T} - 1$ characterizes the distribution of $\mathbf{\Gamma}_1$ for Rician(v)/Rayleigh $(N_\mathrm{T} - v)$ fading. This analysis starts with the singular value decomposition

$$\mathbf{H}_2 = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathcal{H}}, \tag{61}$$

where $\mathbf{U} \doteq N_\mathrm{R} \times N_\mathrm{R}$, $\mathbf{\Sigma} \doteq N_\mathrm{R} \times (N_\mathrm{T} - v)$, and $\mathbf{V} \doteq (N_\mathrm{T} - v) \times (N_\mathrm{T} - v)$. The unitary matrix $\mathbf{U}$, i.e., $\mathbf{U}^{\mathcal{H}}\mathbf{U} = \mathbf{U}\mathbf{U}^{\mathcal{H}} = \mathbf{I}_{N_\mathrm{R}}$, comprises the left singular vectors of $\mathbf{H}_2$. Using the definition of $\mathbf{Q}_2$ from (16) it can be shown that $\mathbf{U}$ is also the matrix with the eigenvectors of $\mathbf{Q}_2$. Further, using (18), we can write the eigendecomposition of $\mathbf{Q}_2$ as:

$$\mathbf{Q}_2 = \mathbf{U}^{\mathcal{H}} \underbrace{\mathrm{diag}(\overbrace{1,\ 1,\ \cdots,\ 1}^{N_v}, \overbrace{0,\ 0,\ \cdots,\ 0}^{N_\mathrm{T}-v})}_{\mathbf{\Lambda} \doteq N_\mathrm{R} \times N_\mathrm{R}} \mathbf{U}. \tag{62}$$

Substituting (62) into (32) yields

$$M_{\mathbf{\Gamma}_1|\mathbf{U}}(\mathbf{\Theta}) = \left| \mathbf{I}_v - \mathbf{\Theta}\left(\mathbf{R}_{\mathrm{T},k}^{11}\right)^{-1} \right|^{-N_v} \times \mathrm{etr}\left(\overbrace{\left[\mathbf{I}_v - \mathbf{\Theta}\left(\mathbf{R}_{\mathrm{T},K}^{11}\right)^{-1}\right]^{-1} \mathbf{\Theta}}^{=\mathbf{\Psi}} \mathbf{M}^{\mathcal{H}}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}}\mathbf{M} \right). \tag{63}$$

Now, averaging the etr$(\cdot)$ term above over $\mathbf{U}$ appears to be tractable only for $\mathbf{H}_{\mathrm{d},2} = \mathbf{0}$, when matrix $\mathbf{U}$ has a known, Haar, distribution [12]. This averaging has been pursued successfully for $v = 1$ in [12]. Herein, we pursue, differently, the more general case whereby $1 \le v < N_\mathrm{T}$. Then,

$$\begin{aligned}\mathbb{E}_{\mathbf{U}}\{\mathrm{etr}(\mathbf{\Psi}\mathbf{M}^{\mathcal{H}}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}}\mathbf{M})\} &= \int_{\mathbb{U}_{N_\mathrm{R}}} \mathrm{etr}(\underbrace{\mathbf{M}\mathbf{\Psi}\mathbf{M}^{\mathcal{H}}}_{=\mathbf{S}}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}})[d\mathbf{U}] \\ &= \int_{\mathbb{U}_{N_\mathrm{R}}} \mathrm{etr}(\mathbf{S}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}})[d\mathbf{U}] \\ &= \int_{\mathbb{U}_{N_\mathrm{R}}} {}_0F_0(\mathbf{S}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}})[d\mathbf{U}].\end{aligned}$$

where $\mathbb{U}_{N_\mathrm{R}}$ is the unitary manifold comprising the $N_\mathrm{R} \times N_\mathrm{R}$ unitary matrices with real diagonal elements, and $[d\mathbf{U}]$ is the normalized Haar invariant probability measure on $\mathbb{U}_{N_\mathrm{R}}$ [10, App. 1]. Matrix $\mathbf{S} \doteq N_\mathrm{R} \times N_\mathrm{R}$, which is given by

$$\mathbf{S} = \mathbf{M}\mathbf{\Psi}\mathbf{M}^{\mathcal{H}} = \mathbf{M}\left[\mathbf{I}_v - \mathbf{\Theta}\left(\mathbf{R}_{\mathrm{T},k}^{11}\right)^{-1}\right]^{-1} \mathbf{\Theta}\mathbf{M}^{\mathcal{H}}, \tag{64}$$

has rank v and distinct nonzero eigenvalues, in general.

Finally, because [24, Eq. (92)], [29, Eq. (4.2)]

$$\int_{\mathbb{U}_{N_\mathrm{R}}} {}_0F_0(\mathbf{S}\mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}})[d\mathbf{U}] = {}_0F_0(\mathbf{S}, \mathbf{\Lambda}), \tag{65}$$

the m.g.f. of the unconditioned $\mathbf{\Gamma}_1$ can be written as

$$M_{\mathbf{\Gamma}_1}(\mathbf{\Theta}) = \left| \mathbf{I}_v - \mathbf{\Theta}\left(\mathbf{R}_{\mathrm{T},k}^{11}\right)^{-1} \right|^{-N_v} {}_0F_0(\mathbf{S}, \mathbf{\Lambda}). \tag{66}$$

B. *Determinantal Expressions for* ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$

Appendix III expresses ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$ as determinant of an $N_\mathrm{R} \times N_\mathrm{R}$ matrix with elementary-function entries, as follows:

- in Appendix III-A, from previous work [10], [25], [29], for when both $\mathbf{S}$, $\mathbf{\Lambda}$ have nonequal eigenvalues, in (88).
- in Appendix III-B, for the more general case when both $\mathbf{S}$, $\mathbf{\Lambda}$ may have equal eigenvalues, in the new expression (91).
- in Appendix III-C, for when $\mathbf{S}$ is rank-v with nonequal nonzero eigenvalues and $\mathbf{\Lambda}$ is idempotent and rank-N_v — as under Rician(v)/Rayleigh$(N_\mathrm{T} - v)$ fading — in the new expression (92). Unfortunately, then, (66) cannot yield SNR m.g.f.s based on the SNR–SC relationship from (21) because the m.g.f. of $\mathbf{\Gamma}_1^{-1}$ could not be deduced from (66).
- in Appendix III-D, for when $\mathbf{S}$ is rank-1 and $\mathbf{\Lambda}$ is idempotent and rank-N—as under Rician(1)/Rayleigh$(N_\mathrm{T} - 1)$ fading—in the new expression (95). This case is considered in more detail below.

C. *New Determinantal Expressions for ZF SNR M.G.F. and AEP for Stream 1 Under Rician(1)/Rayleigh*$(N_\mathrm{T} - 1)$ *Fading*

For Rician(1)/Rayleigh$(N_\mathrm{T} - 1)$ fading, partitioning with $v = 1$ has yielded in (22) the SNR–SC relationship $\gamma_1 = \Gamma_\mathrm{s}\mathbf{\Gamma}_1$, which, along with (66), yields

$$M_{\gamma_1}(s) = M_{\mathbf{\Gamma}_1}(s\Gamma_\mathrm{s}) = (1 - s\Gamma_{K,1})^{-N}\ {}_0F_0(\mathbf{S}, \mathbf{\Lambda}). \tag{67}$$

Appendix III-D expressed ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$ for this case in (95) as

$${}_0F_0(\mathbf{S}, \mathbf{\Lambda}) = A\frac{\Delta_2(N, N_\mathrm{R}, \sigma_1)}{\sigma_1^{N_\mathrm{R}-1}}, \tag{68}$$

where A is defined in (95), and $\Delta_2(N, N_\mathrm{R}, \sigma_1)$ is the determinant of the matrix with elementary-function entries from (96).

Finally, substituting σ_1 from (94) into (68), and the result into (67), yields for the Stream-1 SNR m.g.f. the following new determinantal expression

$$M_{\gamma_1}(s) = A\frac{(1-s\Gamma_{K,1})^{N_\mathrm{T}-2}}{(s\Gamma_{K,1}\alpha)^{N_\mathrm{R}-1}}\Delta_2\left(N, N_\mathrm{R}, \frac{s\Gamma_{K,1}\alpha}{1-s\Gamma_{K,1}}\right), \quad (69)$$

with α defined in (93). Then, substituting (69) into (43) yields the corresponding new ZF AEP expression for Stream 1 under Rician(1)/Rayleigh($N_\mathrm{T}-1$) fading:

$$P_{\mathrm{e},1}^{(70)} = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} A \frac{\left(1+\frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\Gamma_{K,1}\right)^{N_\mathrm{T}-2}}{\left(-\frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\Gamma_{K,1}\alpha\right)^{N_\mathrm{R}-1}} \times \Delta_2\left(N, N_\mathrm{R}, \frac{-\Gamma_{K,1}\alpha\sin^2\frac{\pi}{M}}{\sin^2\theta + \Gamma_{K,1}\sin^2\frac{\pi}{M}}\right) d\theta. \quad (70)$$

The SNR m.g.f. expression (69) and the AEP expression (70) are referenced in Table I, Row 4, for Stream 1.

Remark 4: Under Rician(1)/Rayleigh($N_\mathrm{T}-1$) fading, if the Rayleigh fading is uncorrelated with the Rician fading, the SNRs for the Rayleigh-fading streams, i.e., Streams $i = 2 : N_\mathrm{T}$, can be characterized by viewing this case as Rayleigh($N_\mathrm{T}-1$)/Rician(1) fading that satisfies condition[9] $\mathbf{H}_{\mathrm{d},2} = \mathbf{h}_{\mathrm{d},1}\mathbf{r}_{1,2}^{\mathcal{T}} = \mathbf{0}$. Based on Lemma 1, the SNRs for the Rayleigh-fading streams are then Gamma-distributed as in (37), and their AEPs are described by expression (44) — see Row 2 in Table I. Consequently, the AEP can then be computed for all streams: for the Rician-fading Stream 1 with (70), and for the Rayleigh-fading streams with (44).

D. New Relationship Between ${}_0F_0(\mathbf{S}, \mathbf{\Lambda})$ and ${}_1F_1(N; N_\mathrm{R}; \sigma_1)$, and Ensuing Determinantal Expression for ${}_1F_1(N; N_\mathrm{R}; \sigma_1)$

Corollary 5: If $\mathbf{S}$ and $\mathbf{\Lambda}$ are $N_\mathrm{R}\times N_\mathrm{R}$ matrices, $\mathbf{S}$ of rank 1 with nonzero eigenvalue σ_1 and $\mathbf{\Lambda}$ idempotent of rank N then (59) and (67) reveal the previously unknown relationship

$${}_0F_0(\mathbf{S}, \mathbf{\Lambda}) = {}_1F_1(N; N_\mathrm{R}; \sigma_1). \quad (71)$$

Corollary 6: Eqs. (71) and (68) yield for ${}_1F_1(N; N_\mathrm{R}; \sigma_1)$ the new — determinantal—expression

$${}_1F_1(N; N_\mathrm{R}; \sigma_1) = A\Delta_2(N, N_\mathrm{R}, \sigma_1)/\sigma_1^{N_\mathrm{R}-1}. \quad (72)$$

VII. Numerical Results

A. Description of Settings

For $v = 1$, i.e., the partitioning from (7), Stream-1 AEP results obtained in MATLAB are presented for $N_\mathrm{R} = 4$, $N_\mathrm{T} = 3$, QPSK modulation, and relevant ranges of the per-bit transmit-SNR $\Gamma_\mathrm{b} = \frac{\Gamma_\mathrm{s}}{\log_2 M}$. Matrix $\mathbf{R}_\mathrm{T}$ has been computed as in [9], for a uniform linear antenna array with interelement distance normalized to carrier half-wavelength $d_\mathrm{n} = 1$, Laplacian power azimuth spectrum centered at $\theta_\mathrm{c} = 5°$, and K and AS set to their lognormal-distribution averages for two WINNER II scenarios [9, Table I]:

- B1 (typical urban microcell): $K = 9$ dB, AS $= 3°$, i.e., high transmit-correlation, and, thus, $\mathbf{r}_{2,1} \neq \mathbf{0}$.
- A1 (indoor office/residential): $K = 7$ dB, AS $= 51°$, i.e., low correlation, and, thus, $\mathbf{r}_{2,1} \approx \mathbf{0}$.

For consistency with our previous work in [9], [12], results are shown herein for $v = 1$, $[\mathbf{R}_\mathrm{T}]_{i,i} = 1$, $i = 1 : N_\mathrm{T}$, i.e., for $\mathbb{E}\{|[\mathbf{H}_{\mathrm{r,norm}}]_{i,j}|^2 = 1\}$, $\forall i, j$, and $\mathbf{H}_{\mathrm{d,norm}}$ with arbitrary complex-valued elements.[10] Nevertheless, other (unshown) results have validated our analysis against simulations also for $v > 1$, and $\mathbf{R}_\mathrm{T}$ and $\mathbf{H}_{\mathrm{d,norm}}$ generated as for a MIMO system with distributed transmitters, based on [30].

In our figures, the legends identify results from exact and approximate AEP expressions (with `exact`, `approx`) and from simulation of 10^6 channel and noise samples (with `sim`). All figures depict Rayleigh-only fading, with red lines and markers, and with legend `Ray-Ray`. Additionally, each figure depicts, with black lines and markers, one of the following Rician-fading cases: full-Rician (`Rice-Rice`), Rayleigh–Rician (`Ray-Rice`), or Rician–Rayleigh (`Rice-Ray`), for $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, $\mathbf{h}_{\mathrm{d},1} \approx \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, or $\mathbf{h}_{\mathrm{d},1} \neq \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$. Each case is also identified in figures and discussion by the corresponding row number in Table I.

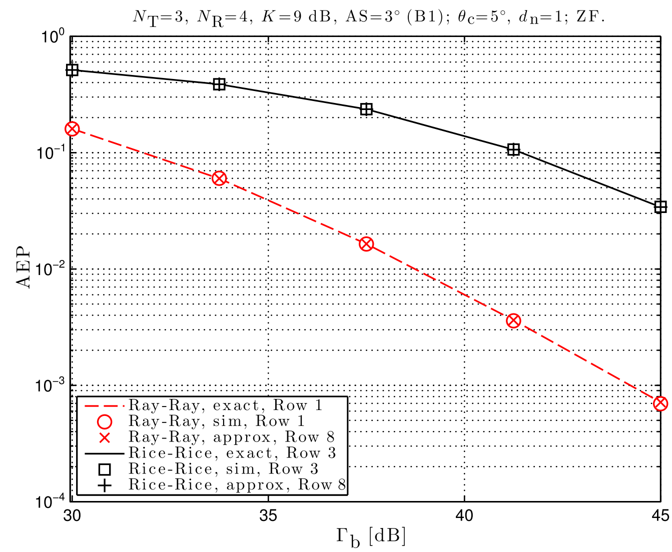

Fig. 1. Stream-1 AEP from exact expression (44), approximate expression (50), and from simulation, for Rayleigh-only fading and for full-Rician fading under condition $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, for QPSK modulation, $N_\mathrm{R} = 4$, $N_\mathrm{T} = 3$, $K = 9$ dB, AS $= 3°$ (i.e., WINNER II scenario B1 averages).

B. Full-Rician Fading, High Correlation, $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$

Fig. 1 depicts full-Rician fading, i.e., $\mathbf{h}_{\mathrm{d},1} \neq \mathbf{0}$, and $\mathbf{H}_{\mathrm{d},2} \neq \mathbf{0}$, under condition $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, which is characterized in Row 3, for scenario B1. Note first that analysis and simulation results agree. Then, as predicted by Corollary 4, the AEP from the exact and approximate expressions agree, because $\mathbf{h}_{\mathrm{d},1} =$

[9]The $1 \times (N_\mathrm{T}-1)$ vector $\mathbf{r}_{1,2}^{\mathcal{T}}$ is analogous of $\mathbf{R}_{2,1}$ from (28).

[10]$\mathbf{R}_\mathrm{T}$ and $\mathbf{H}_{\mathrm{d,norm}}$ were adjusted to satisfy (34), when necessary.

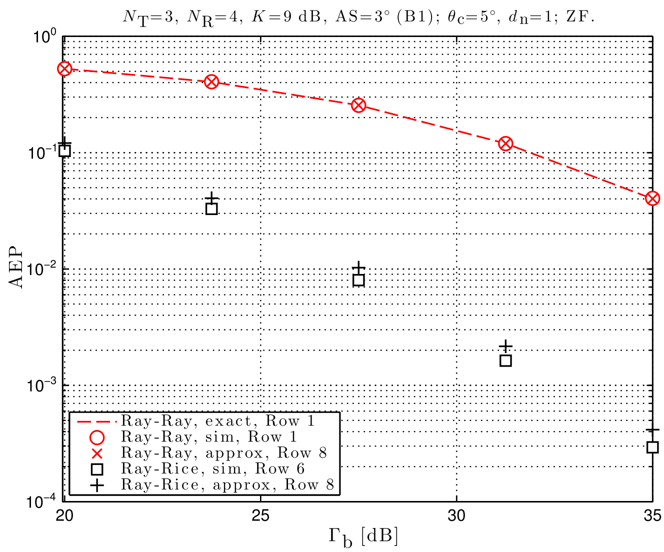

Fig. 2. Stream-1 AEP from exact expression (44), approximate expression (50), and from simulation, for Rayleigh-only fading and for Rayleigh(1)/Rician $(N_{\mathrm{T}}-1)$ fading under conditions $\mathbf{h}_{\mathrm{d},1}=\mathbf{0}$ and $\mathbf{H}_{\mathrm{d},2}\neq\mathbf{0}$, for QPSK modulation, $N_{\mathrm{R}}=4$, $N_{\mathrm{T}}=3$, $K=9$ dB, AS $=3°$ (i.e., WINNER II scenario B1 averages). Since $\mathbf{r}_{2,1}\neq\mathbf{0}$, we have $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$.

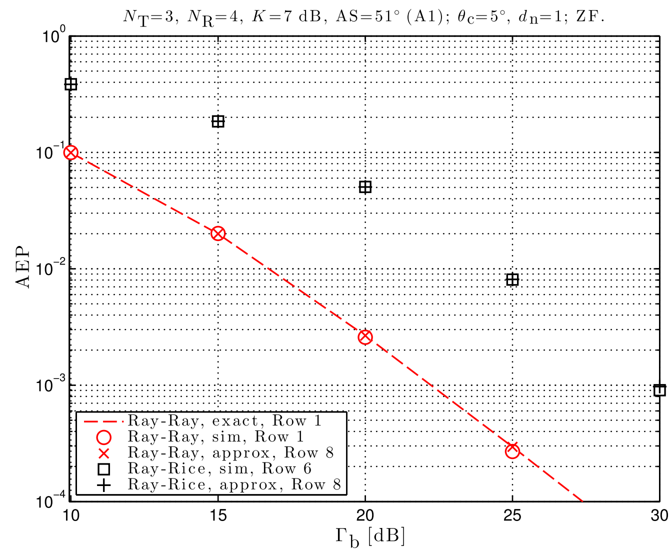

Fig. 3. Stream-1 AEP from exact expression (44), approximate expression (50), and from simulation, for Rayleigh-only fading and for Rayleigh(1)/Rician $(N_{\mathrm{T}}-1)$ fading under conditions $\mathbf{h}_{\mathrm{d},1}=\mathbf{0}$ and $\mathbf{H}_{\mathrm{d},2}\neq\mathbf{0}$, for QPSK modulation, $N_{\mathrm{R}}=4$, $N_{\mathrm{T}}=3$, $K=7$ dB, AS $=51°$ (i.e., WINNER II scenario A1 averages). Since $\mathbf{r}_{2,1}\approx\mathbf{0}$, we have $\mathbf{h}_{\mathrm{d},1}\approx\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$.

$\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$. Finally, as predicted by Corollary 1, Rician fading yields poorer performance than Rayleigh-only fading.

C. Rayleigh–Rician, High Correlation, i.e., $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$

Fig. 2 depicts Rayleigh(1)/Rician$(N_{\mathrm{T}}-1)$ fading with $\mathbf{h}_{\mathrm{d},1}=\mathbf{0}$, $\mathbf{H}_{\mathrm{d},2}\neq\mathbf{0}$, for scenario B1, i.e., $\mathbf{r}_{2,1}\neq\mathbf{0}$, so that $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, which is characterized in Row 6. Since no exact AEP expression is then known, Fig. 2 shows results only from simulation and approximation (see the `Ray-Rice` plots with black □ and + markers), which do not agree because $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$. Further, the plots with black □ and red ○ markers reveal a surprising phenomenon for this fading case, i.e., when the intended stream undergoes Rayleigh fading that is highly-correlated with the interfering fading: Rician-fading interference yields much better performance than Rayleigh-fading interference.

D. Rayleigh–Rician, Low Correlation, i.e., $\mathbf{h}_{\mathrm{d},1}\approx\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$

Fig. 3 depicts the same fading cases as Fig. 2, but for scenario A1, i.e., for low correlation.[11] This yields $\mathbf{r}_{2,1}\approx\mathbf{0}$ and, because $\mathbf{h}_{\mathrm{d},1}=\mathbf{0}$, we have[12] $\mathbf{h}_{\mathrm{d},1}\approx\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, which explains the agreement between the AEP from simulation and the approximate expression for the `Ray-Rice` plots (black markers). Unshown results have confirmed that, for Rayleigh(1)/Rician$(N_{\mathrm{T}}-1)$ fading, the approximate and exact distributions of ZF SNR for Stream 1 become more similar with less correlation between the Rayleigh and Rician fading.

Further, the plots with black □ and red ○ markers reveal the following for this fading case, i.e., when the intended stream undergoes Rayleigh fading that is nearly uncorrelated with

[11]We obtained similar results in [12, Fig. 10].

[12]This case is characterized, approximately, by Row 2.

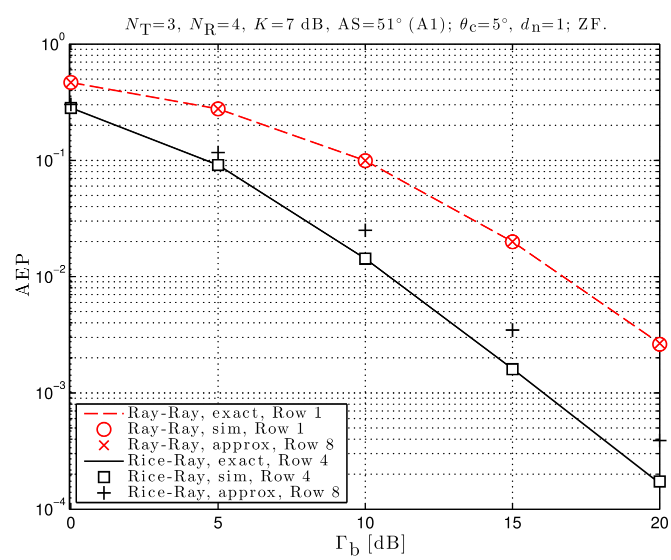

Fig. 4. Stream-1 AEP from exact expressions (44) and (70), approximate expression (50), and from simulation, for Rayleigh-only fading and for Rician(1)/Rayleigh $(N_{\mathrm{T}}-1)$ fading under conditions $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{0}$ and $\mathbf{H}_{\mathrm{d},2}=\mathbf{0}$, i.e., $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, for QPSK modulation, $N_{\mathrm{R}}=4$, $N_{\mathrm{T}}=3$, $K=7$ dB, AS $=51°$ (i.e., WINNER II scenario A1 averages).

the interfering fading: Rician-fading interference yields poorer performance than Rayleigh-fading interference, as predicted by Corollary 1 and Remark 3, as $\mathbf{h}_{\mathrm{d},1}\approx\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$.

E. Rician–Rayleigh, Low Correlation, $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$

Fig. 4 depicts Rician(1)/Rayleigh$(N_{\mathrm{T}}-1)$ fading, i.e., $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{0}$ and $\mathbf{H}_{\mathrm{d},2}=\mathbf{0}$, for scenario A1. This case implies $\mathbf{h}_{\mathrm{d},1}\neq\mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$, and is characterized in Row 4. The new exact determinantal AEP expression (70) agrees with the simulation results, but not with the approximate AEP expression (50), which is explained by Corollary 4, as $\mathbf{h}_{\mathrm{d},1} \neq \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$.

F. Condition $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$ *Impact on Relative Performance*

The relative positions of plots with black vs. red lines in the figures reveal that if condition $\mathbf{h}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{r}_{2,1}$ holds then Rayleigh-only fading outperforms Rician fading, e.g., in Figs. 1 and 3 — which is supported by Corollary 1 and Remark 3—and also *vice versa*, e.g., in Figs. 2 and 4.

VIII. Summary and Conclusion

By characterizing the distribution of the matrix-SC in the NCWD Gramian matrix induced by a nonzero-mean Gaussian matrix, we analyzed MIMO ZF under transmit-correlated Rician fading. Although expressing the m.g.f. of the unconditioned matrix-SC (and the ZF SNRs) remains intractable for general Rician fading, we have succeeded for two cases.

The first tractable case arose by imposing the mean-correlation condition that yields a CWD for the matrix-SC. We have shown that this condition also renders exact a previously-proposed approximation with the Gamma distribution of the unknown distribution of the ZF SNRs under Rician fading. This finding has corroborated previous observations made in our work, and explained accuracy inconsistencies observed for the fading case usually assumed by others.

The second tractable case is that of Rician–Rayleigh fading. Then, for the matrix-SC m.g.f., we have derived new expressions in terms of the determinant of a matrix with elementary-function entries. Thus, we have also obtained new, determinantal expressions for the ZF SNR m.g.f. and AEP. Finally, we have revealed new determinantal expressions for, and a new relationship between, hypergeometric functions of matrix and scalar arguments.

Numerical results have confirmed analysis predictions, i.e., that: 1) the previously-proposed approximation becomes exact under the newly-discovered condition; 2) Rician-fading streams may still experience Rayleigh-like SNR distributions, under the newly-discovered condition; 3) the condition also determines the relative performance with Rician vs. Rayleigh-only fading. Finally, numerical results have also revealed a surprising phenomenon when the intended stream undergoes Rayleigh fading and the intended and interfering fading are highly correlated: Rician-fading interference can then greatly benefit performance vs. Rayleigh-fading interference.

Appendix I
Further Results on Condition $\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}$

Recall that, for $\mathbf{H} \sim \mathcal{CN}(\mathbf{H}_\mathrm{d}, \mathbf{I}_{N_\mathrm{R}} \otimes \mathbf{R}_{\mathrm{T},K}^{\mathcal{H}})$, $\mathbf{R}_{\mathrm{T},K}$ is the covariance matrix of the columns of $\mathbf{H}^{\mathcal{H}}$. Using the UL decomposition of $\mathbf{R}_{\mathrm{T},K} = \mathbf{A}\mathbf{A}^{\mathcal{H}}$, and defining $\mathbf{H}_\mathrm{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_\mathrm{R}} \otimes \mathbf{I}_{N_\mathrm{T}})$, we can write

$$\mathbf{H} = \mathbf{H}_\mathrm{d} + \mathbf{H}_\mathrm{w}\mathbf{A}^{\mathcal{H}}, \tag{73}$$

so that

$$\mathbf{H}\mathbf{A}^{-\mathcal{H}} = \mathbf{H}_\mathrm{d}\mathbf{A}^{-\mathcal{H}} + \mathbf{H}_\mathrm{w}. \tag{74}$$

Based on the partitionings of $\mathbf{H}_\mathrm{d}$ and $\mathbf{A}^{-\mathcal{H}}$, we can write

$$\mathbf{H}_\mathrm{d}\mathbf{A}^{-\mathcal{H}} = \begin{pmatrix}\mathbf{H}_{\mathrm{d},1}\mathbf{A}^{11,\mathcal{H}} + \mathbf{H}_{\mathrm{d},2}\mathbf{A}^{12,\mathcal{H}} & \mathbf{H}_{\mathrm{d},2}\mathbf{A}^{22,\mathcal{H}}\end{pmatrix},$$

which, based on (9)–(11) and (28), becomes

$$\mathbf{H}_\mathrm{d}\mathbf{A}^{-\mathcal{H}} = \begin{pmatrix}[\mathbf{H}_{\mathrm{d},1} - \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}]\,\mathbf{A}^{11,\mathcal{H}} & \mathbf{H}_{\mathrm{d},2}\mathbf{A}^{22,\mathcal{H}}\end{pmatrix}. \tag{75}$$

Finally, (34), (74), and (75) prove the following Lemma.

Lemma 3:

$$\begin{aligned}&\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}\\ &\quad\Leftrightarrow \mathbf{H}\mathbf{A}^{-\mathcal{H}} = \begin{pmatrix}\mathbf{0} & \mathbf{H}_{\mathrm{d},2}\mathbf{A}^{22,\mathcal{H}}\end{pmatrix} + \begin{pmatrix}\mathbf{H}_{\mathrm{w},1} & \mathbf{H}_{\mathrm{w},2}\end{pmatrix},\end{aligned} \tag{76}$$

i.e., the mean-correlation condition is equivalent with the fact that canceling the transmit-correlation in the channel matrix yields a matrix whose first v columns are zero-mean.

The following corollary summarizes from Theorem 1 and Lemma 3 the necessary and sufficient conditions for $\mathbf{\Gamma}_1$ to be CWD.

Corollary 7:

$$\begin{aligned}&\mathbf{\Gamma}_1 \sim \mathcal{CW}_v\left(N_v, \left(\mathbf{R}_{\mathrm{T},k}^{11}\right)^{-1}\right)\\ &\Leftrightarrow \mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}\\ &\Leftrightarrow \mathbf{H}\mathbf{A}^{-\mathcal{H}} = \begin{pmatrix}\mathbf{0} \; \mathbf{H}_{\mathrm{d},2}\mathbf{A}^{22,\mathcal{H}}\end{pmatrix} + \begin{pmatrix}\mathbf{H}_{\mathrm{w},1} \; \mathbf{H}_{\mathrm{w},2}\end{pmatrix}.\end{aligned}$$

Corollary 8 (Mean-Correlation "Parallelism"): For nonsingular $\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}$,

$$\begin{aligned}\mathbf{H}_{\mathrm{d},1} &= \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}\\ &\Rightarrow \left(\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)^{-1}\left(\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}\right)\\ &= \left(\mathbb{E}\left\{\mathbf{H}_{\mathrm{r},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{r},2}\right\}\right)^{-1}\left(\mathbb{E}\left\{\mathbf{H}_{\mathrm{r},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{r},1}\right\}\right).\end{aligned} \tag{77}$$

Proof: Follows by premultiplying $\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{R}_{\mathrm{T},K_{21}}$ with $\left(\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}$, and expressing $\mathbf{R}_{\mathrm{T},K_{22}}$ and $\mathbf{R}_{\mathrm{T},K_{21}}$ from (6).

Appendix II
Proof of Theorem 2: $\mathbf{H}_{\mathrm{d},1} = \mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1} \Leftrightarrow (\widehat{\mathbf{R}}_{\mathrm{T},K}^{11})^{-1} = (\mathbf{R}_{\mathrm{T},K}^{11})^{-1}$

Let us first find a simpler condition equivalent with $\left(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11}\right)^{-1} = \left(\mathbf{R}_{\mathrm{T},K}^{11}\right)^{-1}$. Equalizing the SC representation for $\left(\mathbf{R}_{\mathrm{T},K}^{11}\right)^{-1}$ from (13) with that obtained analogously for $\left(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11}\right)^{-1}$ based on (46) yields

$$\begin{aligned}&\cancel{\mathbf{R}_{\mathrm{T},K11}} - \mathbf{R}_{\mathrm{T},K_{12}}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{R}_{\mathrm{T},K_{21}} = \cancel{\mathbf{R}_{\mathrm{T},K_{11}}} + \frac{1}{N_\mathrm{R}}\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}\\ &\quad - \left(\mathbf{R}_{T,K_{12}} + \frac{1}{N_\mathrm{R}}\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)\underbrace{\left(\mathbf{R}_{\mathrm{T},K_{22}} + \frac{1}{N_\mathrm{R}}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)^{-1}}_{=\mathbf{P}}\\ &\quad\times\left(\mathbf{R}_{\mathrm{T},K_{21}} + \frac{1}{N_\mathrm{R}}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}\right),\end{aligned} \tag{78}$$

i.e.,

$$\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}+\mathbf{R}_{\mathrm{T},K_{12}}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{R}_{\mathrm{T},K_{21}} =\left(\mathbf{R}_{\mathrm{T},K_{12}}+\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)\mathbf{P}\left(\mathbf{R}_{\mathrm{T},K_{21}}+\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}\right),$$

or

$$\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\overbrace{\left(\mathbf{I}_{N_{\mathrm{R}}}-\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}\mathbf{P}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\right)}^{=\mathbf{Q}}\mathbf{H}_{\mathrm{d},1} + N_{\mathrm{R}}\mathbf{R}_{\mathrm{T},K_{12}}\left(\mathbf{R}_{\mathrm{T},K_{22}}^{-1}-\mathbf{P}\right)\mathbf{R}_{\mathrm{T},K_{21}} = \mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\overbrace{\mathbf{H}_{\mathrm{d},2}\mathbf{P}\mathbf{R}_{\mathrm{T},K_{21}}}^{=\mathbf{F}}+\overbrace{\mathbf{R}_{\mathrm{T},K_{12}}\mathbf{P}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}}^{=\mathbf{F}^{\mathcal{H}}}\mathbf{H}_{\mathrm{d},1},$$

or, finally,

$$\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},1}+N_{\mathrm{R}}\mathbf{R}_{\mathrm{T},K_{12}}\left(\mathbf{R}_{\mathrm{T},K_{12}}^{-1}-\mathbf{P}\right)\mathbf{R}_{\mathrm{T},K_{21}} = \mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{F}+\mathbf{F}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}. \quad (79)$$

The Woodbury matrix-inversion formula [19, p. 165] yields

$$\begin{aligned}\mathbf{Q} &=\mathbf{I}_{N_{\mathrm{R}}}-\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}\left(\mathbf{R}_{\mathrm{T},K_{22}}+\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\\ &=\left(\mathbf{I}_{N_{\mathrm{R}}}+\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\right)^{-1}\\ \mathbf{P} &=\left(\mathbf{R}_{\mathrm{T},K_{22}}+\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},2}\right)^{-1}\\ &=\mathbf{R}_{\mathrm{T},K_{22}}^{-1}-\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\\ &\quad\times\frac{1}{N_{\mathrm{R}}}\left(\mathbf{I}_{N_{\mathrm{R}}}+\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\right)^{-1}\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}, \quad (80)\end{aligned}$$

i.e.,

$$\mathbf{R}_{\mathrm{T},K_{22}}^{-1}-\mathbf{P}=\frac{1}{N_{\mathrm{R}}}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}. \quad (81)$$

Substituting (81) into (79) yields

$$\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},1}+\overbrace{\mathbf{R}_{\mathrm{T},K_{12}}\mathbf{R}_{\mathrm{T},K_{22}}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}}^{=\mathbf{B}^{\mathcal{H}}}\mathbf{Q}\overbrace{\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{R}_{\mathrm{T},K_{21}}}^{=\mathbf{B}} = \mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{F}+\mathbf{F}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1},$$

or

$$\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},1}-\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{F}-\mathbf{F}^{\mathcal{H}}\mathbf{H}_{\mathrm{d},1}+\mathbf{B}^{\mathcal{H}}\mathbf{Q}\mathbf{B}=\mathbf{0}, \quad (82)$$

where

$$\begin{aligned}\mathbf{F} &=\mathbf{H}_{\mathrm{d},2}\mathbf{P}\mathbf{R}_{\mathrm{T},K_{21}}\\ &=\mathbf{H}_{\mathrm{d},2}\left(R_{\mathrm{T},K_{22}}^{-1}-\frac{1}{N_{\mathrm{R}}}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\right)\mathbf{R}_{\mathrm{T},K_{21}}\\ &=\mathbf{B}-\underbrace{\frac{1}{N_{\mathrm{R}}}\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{H}_{\mathrm{d},2}^{\mathcal{H}}}_{\overset{(80)}{=}\mathbf{Q}^{-1}-\mathbf{I}_{N_{\mathrm{R}}}}\mathbf{Q}\mathbf{B}\\ &=\mathbf{B}-(\mathbf{Q}^{-1}-\mathbf{I}_{N_{\mathrm{R}}})\mathbf{Q}\mathbf{B}=\mathbf{Q}\mathbf{B}.\end{aligned}$$

Thus, (82) becomes

$$\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},1}-\mathbf{H}_{\mathrm{d},1}^{\mathcal{H}}\mathbf{Q}\mathbf{B}-\mathbf{B}^{\mathcal{H}}\mathbf{Q}\mathbf{H}_{\mathrm{d},1}+\mathbf{B}^{\mathcal{H}}\mathbf{Q}\mathbf{B}=\mathbf{0}, \quad (83)$$

which is the sought simpler expression equivalent with $(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11})^{-1}=(\mathbf{R}_{\mathrm{T},K}^{11})^{-1}$.

Now, let us assume that $(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11})^{-1}=(\mathbf{R}_{\mathrm{T},K}^{11})^{-1}$ holds, i.e., that (83) holds. Then, with $\widetilde{\mathbf{H}}_{\mathrm{d},1}=\mathbf{Q}^{1/2}\mathbf{H}_{\mathrm{d},1}$ and $\widetilde{\mathbf{B}}=\mathbf{Q}^{1/2}\mathbf{B}$, (83) becomes

$$\widetilde{\mathbf{H}}_{\mathrm{d},1}^{\mathcal{H}}\widetilde{\mathbf{H}}_{\mathrm{d},1}-\widetilde{\mathbf{H}}_{\mathrm{d},1}^{\mathcal{H}}\widetilde{\mathbf{B}}-\widetilde{\mathbf{B}}^{\mathcal{H}}\widetilde{\mathbf{H}}_{\mathrm{d},1}+\widetilde{\mathbf{B}}^{\mathcal{H}}\widetilde{\mathbf{B}}=\mathbf{0}, \quad (84)$$

which can be written further as

$$\widetilde{\mathbf{H}}_{\mathrm{d},1}^{\mathcal{H}}(\widetilde{\mathbf{H}}_{\mathrm{d},1}-\widetilde{\mathbf{B}})-\widetilde{\mathbf{B}}^{\mathcal{H}}(\widetilde{\mathbf{H}}_{\mathrm{d},1}-\widetilde{\mathbf{B}})=\mathbf{0}, \quad (85)$$

or

$$(\widetilde{\mathbf{H}}_{\mathrm{d},1}-\widetilde{\mathbf{B}})^{\mathcal{H}}(\widetilde{\mathbf{H}}_{\mathrm{d},1}-\widetilde{\mathbf{B}})=\mathbf{0}, \quad (86)$$

which implies

$$\widetilde{\mathbf{H}}_{\mathrm{d},1}=\widetilde{\mathbf{B}}\Leftrightarrow\mathbf{H}_{\mathrm{d},1}=\mathbf{B}=\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{R}_{\mathrm{T},K_{21}}=\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{2,1}.$$

Assuming, conversely, that $\mathbf{H}_{\mathrm{d},1}=\mathbf{H}_{\mathrm{d},2}\mathbf{R}_{\mathrm{T},K_{22}}^{-1}\mathbf{R}_{\mathrm{T},K_{21}}$ implies that $\mathbf{H}_{\mathrm{d},1}=\mathbf{B}$, which reduces the left-hand side of (83) to $\mathbf{0}$, and implies $(\widehat{\mathbf{R}}_{\mathrm{T},K}^{11})^{-1}=(\mathbf{R}_{\mathrm{T},K}^{11})^{-1}$.

Appendix III
Determinantal Expressions for ${}_0F_0(\mathbf{S},\mathbf{\Lambda})$

A. Expression for When Both $\mathbf{S}$, $\mathbf{\Lambda}$ Have Distinct Eigenvalues

Given $\sigma_1>\sigma_2>\cdots>\sigma_{N_{\mathrm{R}}}$ and $\lambda_1>\lambda_2>\cdots>\lambda_{N_{\mathrm{R}}}$, let us define

$$\begin{aligned}g(\boldsymbol{\sigma},\boldsymbol{\lambda}) &=g\left(\sigma_1,\cdots,\sigma_{N_{\mathrm{R}}},\lambda_1,\cdots,\lambda_{N_{\mathrm{R}}}\right)\\ &=\frac{\det\left(e^{\sigma_i\lambda_j}\right)}{\prod_{i<j}(\sigma_i-\sigma_j)\prod_{i<j}(\lambda_i-\lambda_j)},\end{aligned} \quad (87)$$

where $\det(e^{\sigma_i\lambda_j})$ is the determinant of the $N_{\mathrm{R}}\times N_{\mathrm{R}}$ matrix with elements $[\mathbf{D}]_{i,j}=e^{\sigma_i\lambda_j}$, $i,j=1:N_{\mathrm{R}}$.

Lemma 4 ([10], [25], [29]): If $N_{\mathrm{R}}\times N_{\mathrm{R}}$ matrices $\mathbf{S}$ and $\mathbf{\Lambda}$ both have distinct eigenvalues, i.e., $\sigma_1>\sigma_2>\cdots>\sigma_{N_{\mathrm{R}}}$ and $\lambda_1>\lambda_2>\cdots>\lambda_{N_{\mathrm{R}}}$, then

$${}_0F_0(\mathbf{S},\mathbf{\Lambda})=g(\boldsymbol{\sigma},\boldsymbol{\lambda})\phi(N_{\mathrm{R}}), \quad (88)$$

where $\phi(N_{\mathrm{R}})=\prod_{j=1}^{N_{\mathrm{R}}}(j-1)!$.

B. New Expression for When Both $\mathbf{S}$, $\mathbf{\Lambda}$ May Have Non-Distinct Eigenvalues

Let the distinct eigenvalues of $\mathbf{S}$ and $\mathbf{\Lambda}$ be ordered as follows

$$\sigma_{(1)}^0>\sigma_{(2)}^0>\cdots>\sigma_{(m_{\mathcal{S}})}^0, \quad (89)$$

$$\lambda_{(1)}^0>\lambda_{(2)}^0>\cdots>\lambda_{(m'_{\mathcal{L}})}^0. \quad (90)$$

The multiplicity of $\sigma_{(i)}^0$ is denoted with m_i, $i=1:\mathcal{S}$. The multiplicity of $\lambda_{(i)}^0$ is denoted with m'_i, $i=1:\mathcal{L}$. Let $\boldsymbol{\sigma}^0$ be the vector with $\sigma^0_{(1)}, \sigma^0_{(2)}, \ldots, \sigma^0_{(m_{\mathcal{S}})}$ repeated according to their multiplicities. Let $\boldsymbol{\lambda}^0$ be the vector with $\lambda^0_{(1)}, \lambda^0_{(2)}, \ldots, \lambda^0_{(m_{\mathcal{S}})}$ repeated according to their multiplicities. Finally, define

$$\begin{aligned} a_i &= m_1 - i, && \text{for } 1 \le i \le m_1,\\ a_i &= \sum\nolimits_{p=1}^{k+1} m_p - i, && \text{for } \sum_{p=1}^{k} m_p < i \le \sum_{p=1}^{k+1} m_p,\\ b_j &= m'_1 - j, && \text{for } 1 \le j \le m'_1,\\ b_j &= \sum\nolimits_{p=1}^{k+1} m'_p - j, && \text{for } \sum_{p=1}^{k} m'_p < j \le \sum_{p=1}^{k+1} m'_p. \end{aligned}$$

Lemma 5: The continuous extension of $g(\boldsymbol{\sigma}, \boldsymbol{\lambda})$ from (87) at $(\boldsymbol{\sigma}^0, \boldsymbol{\lambda}^0)$ helps express ${}_0F_0(\mathbf{S}, \boldsymbol{\Lambda})$ from (88), when both $\mathbf{S}$ and $\boldsymbol{\Lambda}$ may have equal eigenvalues, as

$$\frac{\det\left(\left. \frac{\partial^{a_i+b_j}\left(e^{\sigma_i \lambda_j}\right)}{\partial \sigma_i^{a_i} \partial \lambda_j^{b_j}} \right|_{\substack{\sigma_i = [\boldsymbol{\sigma}^0]_i \\ \lambda_j = [\boldsymbol{\lambda}^0]_j}} \right) \frac{\phi(N_{\mathrm{R}})}{\prod_{i=1}^{\mathcal{S}} \phi(m_i) \prod_{i=1}^{\mathcal{L}} \phi(m'_i)}}{\prod_{i<j}^{\mathcal{S}} (\sigma^0_{(i)} - \sigma^0_{(j)})^{m_i m_j} \prod_{i<j}^{\mathcal{L}} \left(\lambda^0_{(i)} - \lambda^0_{(j)}\right)^{m'_i m'_j}}. \quad (91)$$

Proof: Follows by generalizing [25, Lemma 2].

Expression (91) reduces to previously derived expressions:

- [25, Eq. (10)], for both $\mathbf{S}$ and $\boldsymbol{\Lambda}$ with distinct eigenvalues — see also (88).
- [25, Eq. (16)], for $\mathbf{S}$ with distinct eigenvalues and $\boldsymbol{\Lambda}$ with one subset of equal eigenvalues.
- [25, Eq. (18)], for $\mathbf{S}$ with distinct eigenvalues and $\boldsymbol{\Lambda}$ with one subset of zero eigenvalues.

C. New Expression for When $\mathbf{S}$ is Rank-v With Distinct Nonzero Eigenvalues, and $\boldsymbol{\Lambda}$ is Rank-N_v Idempotent

Corollary 9: If $\mathbf{S}$ and $\boldsymbol{\Lambda}$ are $N_{\mathrm{R}} \times N_{\mathrm{R}}$ matrices, $\mathbf{S}$ of rank v and with the nonzero distinct eigenvalues σ_i, $i = 1:v$, and $\boldsymbol{\Lambda}$ of rank N_v and idempotent, then ${}_0F_0(\mathbf{S}, \boldsymbol{\Lambda})$ is given by

$$\frac{\Delta_1(N_v, N_{\mathrm{R}}, \mathbf{S})}{\prod_{i=1}^{v} \sigma_i^{N_{\mathrm{R}}-v} \prod_{i<j}^{v} (\sigma_i - \sigma_j)} \frac{\phi(N_{\mathrm{R}})}{\phi(N_{\mathrm{R}} - v)\phi(N_{\mathrm{R}} - N_v)\phi(N_v)} \quad (92)$$

where $\Delta_1(N_v, N_{\mathrm{R}}, \mathbf{S})$ is the determinant of the $N_{\mathrm{R}} \times N_{\mathrm{R}}$ matrix with (elementary-function) elements

$$\begin{cases} e^{\sigma_i} \sigma_i^{N_v - j}, & \text{if } i \le v, j \le N_v \\ \sigma_i^{N_{\mathrm{R}} - j}, & \text{if } i \le v, j > N_v \\ (N_v - j)! \binom{N_{\mathrm{R}} - i}{N_v - j}, & \text{if } i > v, j \le N_v, N_{\mathrm{R}} - i \ge N_v - j \\ 0, & \text{if } i > v, j \le N_v, N_{\mathrm{R}} - i < N_v - j \\ (N_{\mathrm{R}} - i)!, & \text{if } i > v, j > N_v, i = j \\ 0, & \text{if } i > v, j > N_v, i \ne j. \end{cases}$$

Proof: Follows from (91).

Substituting (92) into (66) yields the first known expression (in terms of the determinant of a matrix whose entries are elementary functions) for the m.g.f. of $\boldsymbol{\Gamma}_1$, i.e., for the SC in the NCWD Gramian matrix $\mathbf{W} = \mathbf{H}^{\mathcal{H}}\mathbf{H}$ obtained from matrix $\mathbf{H} = (\mathbf{H}_1 \quad \mathbf{H}_2)$ with mean $(\mathbf{H}_{\mathrm{d},1} \quad \mathbf{0})$.

D. New Expression for When $\mathbf{S}$ is Rank-1, and $\boldsymbol{\Lambda}$ is Rank-N Idempotent

For $v = 1$, N_v reduces to $N_{\mathrm{R}} - N_{\mathrm{T}} + 1 = N$, matrix $\mathbf{M} \doteq N_{\mathrm{R}} \times v$ reduces to vector $\boldsymbol{\mu} \doteq N_{\mathrm{R}} \times 1$, and $\mathbf{S}$ can be written from (64) as follows[13]:

$$\begin{aligned} \mathbf{S} &= \frac{s\Gamma_{\mathrm{s}}}{1 - s\Gamma_{\mathrm{s}}/\mathbf{R}^{11}_{\mathrm{T},K}} \boldsymbol{\mu}\boldsymbol{\mu}^{\mathcal{H}} \\ &= \frac{s\,\overbrace{\Gamma_{\mathrm{s}}/\mathbf{R}^{11}_{\mathrm{T},K}}^{\Gamma_{K,1}}}{1 - s\Gamma_{\mathrm{s}}/\mathbf{R}^{11}_{\mathrm{T},K}} \overbrace{\mathbf{R}^{11}_{\mathrm{T},K} \|\boldsymbol{\mu}\|^2}^{=\alpha} \frac{\boldsymbol{\mu}}{\|\boldsymbol{\mu}\|} \frac{\boldsymbol{\mu}^{\mathcal{H}}}{\|\boldsymbol{\mu}\|}, \end{aligned} \quad (93)$$

i.e., $\mathbf{S}$ is rank-1 and with the nonzero eigenvalue given by

$$\sigma_1 = \frac{s\Gamma_{K,1}}{1 - s\Gamma_{K,1}} \alpha. \quad (94)$$

Lemma 6: If $\mathbf{S}$ and $\boldsymbol{\Lambda}$ are $N_{\mathrm{R}} \times N_{\mathrm{R}}$ matrices, $\mathbf{S}$ of rank 1 with nonzero eigenvalue σ_1, and $\boldsymbol{\Lambda}$ of rank N and idempotent, then ${}_0F_0(\mathbf{S}, \boldsymbol{\Lambda})$ is given by

$${}_0F_0(\mathbf{S}, \boldsymbol{\Lambda}) = \underbrace{\frac{(N_{\mathrm{R}} - 1)!}{\phi(N)\phi(N_{\mathrm{R}} - N)}}_{=A} \frac{\Delta_2(N, N_{\mathrm{R}}, \sigma_1)}{\sigma_1^{N_{\mathrm{R}}-1}}, \quad (95)$$

where $\Delta_2(N, N_{\mathrm{R}}, \sigma_1)$ is the determinant of the $N_{\mathrm{R}} \times N_{\mathrm{R}}$ matrix with (elementary-function) elements

$$\begin{cases} e^{\sigma_1} \sigma_1^{N - j}, & \text{if } i = 1, j \le N \\ \sigma_1^{N_{\mathrm{R}} - j}, & \text{if } i = 1, j > N \\ (N - j)! \binom{N_{\mathrm{R}} - i}{N - j}, & \text{if } i > 1, j \le N, N_{\mathrm{R}} - i \ge N - j \\ 0, & \text{if } i > 1, j \le N, N_{\mathrm{R}} - i < N - j \\ (N_{\mathrm{R}} - i)!, & \text{if } i > 1, j > N, i = j \\ 0, & \text{if } i > 1, j > N, i \ne j. \end{cases} \quad (96)$$

Proof: Follows from (92).

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[13] Here, we replace matrix symbol $\mathbf{M}$ with vector symbol $\boldsymbol{\mu}$ used in [12].

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Constantin (Costi) Siriteanu was born in Sibiu, Romania. He received the Bachelor's and Master's degrees in control systems from "Gheorghe Asachi" Technical University, Iasi, Romania, in 1995 and 1996, respectively, and the Ph.D. degree in electrical and computer engineering from Queen's University, Kingston, ON, Canada, in 2006. His Ph.D. thesis was on the performance-complexity tradeoff for smart antennas. Between September 2006 and March 2014 he worked as Researcher and Assistant Professor in Korea (Seoul National University, Kyung Hee Univesity, Hanyang University), Canada (Queen's University), and Japan (Hokkaido University, University of Tokyo). Since April 2014, he is a CAREN Specially-Appointed Assistant Professor with the Graduate School of Information Science and Technology, Osaka University. His research interests have been in developing multivariate statistics concepts that help analyze and evaluate the performance of multiple-input/multiple-output (MIMO) wireless communications systems under realistic statistical assumptions about channel fading. He has recently been working on applications of computer algebra to the deduction of implicit representations of MIMO performance measures (i.e., as solutions of differential equations).

Akimichi Takemura received the B.A. degree in economics and the M.A. degree in statistics from University of Tokyo, Tokyo, Japan, in 1976 and 1978, respectively and the Ph.D. degree in statistics from Stanford University, Standford, CA, USA, in 1982. He was an acting Assistant Professor with the Department of Statistics, Stanford University, from September 1992 to June 1983, and a visiting Assistant Professor with the Department of Statistics, Purdue University, from September 1983 to May 1984. In June 1984, he has joined University of Tokyo, where he has been a Professor of Statistics with the Department of Mathematical Informatics since April 2001. He served as President of the Japan Statistical Society from January 2011 to June 2013. He has been working on multivariate distribution theory in statistics. Currently, his main area of research is algebraic statistics. He also works on game-theoretic probability, which is a new approach to probability theory.

Satoshi Kuriki received the bachelor's and Ph.D. degrees from University of Tokyo, Tokyo, Japan, in 1982 and 1993, respectively. He is a Professor with the Institute of Statistical Mathematics (ISM), Tokyo, Japan, where he is also serving as Director of the Department of Mathematical Analysis and Statistical Inference. His current major research interests include geometry of random fields, multivariate analysis, multiple comparisons, graphical models, optimal designs, and genetic statistics.

Donald St. P. Richards received the B.Sc. (special honors) degree with first-class honors and the Ph.D. degree in mathematical statistics, both from the University of the West Indies, Kingston, Jamaica, in 1976 and 1978, respectively. He is a Professor with the Department of Statistics, Pennsylvania State University, where his research focuses on multivariate statistical analysis, harmonic analysis and special functions, and applied probability. Prior to joining Pennsylvania State University in 2002, he was a faculty member at the University of the West Indies (1979–1981), the University of North Carolina (1981–1987), and the University of Virginia (1987–2002). He has also held numerous visiting appointments, including: a visiting faculty member at the University of Wyoming (Laramie, WY, USA), a Member of the Institute for Advanced Study (Princeton, NJ, USA), Visiting Scientist at the Statistical and Applied Mathematical Sciences Institute (Research Triangle Park, NC, USA), the Institute of Statistical Mathematics (Tachikawa, Japan), and Visiting Professor at the University of Heidelberg (Heidelberg, Germany). He was elected Fellow of the Institute of Mathematical Statistics in 1999 and Fellow of the American Mathematical Society in 2013.

Hyungdong Shin (S'01–M'04–SM'11) received the B.S. degree in electronics engineering from Kyung Hee University, Seoul, Korea, in 1999, and the M.S. and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 2001 and 2004, respectively. During his postdoctoral research at the Massachusetts Institute of Technology (MIT) from 2004 to 2006, he was with the Wireless Communication and Network Sciences Laboratory within the Laboratory for Information Decision Systems (LIDS). In 2006, he joined Kyung Hee University, Seoul, Korea, where he is now an Associate Professor at the Department of Electronics and Radio Engineering. His research interests include wireless communications and information theory with current emphasis on MIMO systems, cooperative and cognitive communications, network interference, vehicular communication networks, location-aware radios and networks, physical-layer security, molecular communications. He was honored with the Knowledge Creation Award in the field of computer science from the Korean Ministry of Education, Science and Technology (2010). He received the IEEE Communications Society Guglielmo Marconi Prize Paper Award (2008) and William R. Bennett Prize Paper Award (2012). He served as a Technical Program Co-chair for the IEEE WCNC (2009 PHY Track) and the IEEE Globecom (Communication Theory Symposium, 2012). He was an Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS (2007–2012). He is currently an Editor for IEEE COMMUNICATIONS LETTERS.